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LAMBDA SINGLE-PARTICLE ENERGY LEVELS IN ${}^{139}_{\Lambda}La$ BY USING NUMEROV METHOD

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Abstract

In this research paper, the Λ single particle energies and radial wave functions of ${}^{139}_{\Lambda}La$ have been investigated, by solving numerically Schrödinger equation with Numerov Method. The phenomenological Woods-Saxon central potential and Woods-Saxon potential including spin orbit term are used in this calculation. The root-mean square distances between Λ and core nucleus ${}^{139}_{\Lambda}La$ were calculated with normalized wave function. In this calculation, Λ single particle energy levels of ${}^{139}_{\Lambda}La$ are $1s_{1/2}$, $1p_{1/2}$, $1p_{3/2}$, 1d3/2, $1d_{5/2}$, $1f_{5/2}$, 1f7/2, $1g_{9/2}$, $2s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$. This calculated results are agree with H.Bando calculated results and slightly different from experimental results.

Keywords: Numerov method, Behaviour of wave functions and energy levels, Lambda hyper nuclei.

Introduction

Hyperon and Hypernucleus

Hyperons are special class of baryons and consisting of one or more strange quarks. All hyperons are fermions and they have half-integer spin. Hyperons are unstable particles and heavier than nucleons. They have the life time of the order of 10^{-10}s and they decay weakly into nucleons and light particles such as π-mesons, electrons and neutrinos. Their formation time is 10^{-23}s which is typical for strong interaction.

Various types of hyperons are lambda (Λ^0), sigma (Σ^-, Σ^0, Σ^+), xi (Ξ^-, Ξ^0) and omega (Ω). All hyperons have a spin of (1/2), but omega has a spin of (3/2). The Λ hyperon is an isospin singlet of strangeness -1. The Σ hyperon occurs as an isospin triplet of baryons Σ^-, Σ^0, Σ^+ and they have strangeness -1. The cascade particles xi (Ξ^-, Ξ^0) have strangeness -2. The omega-minus (Ω) is strangeness -3 because it is composed of three strange quarks.

The lambda hyperon Λ^0 was firstly discovered in October 1950, by V.D.Hopper and S. Biswas of the University of Melbourne, as a neutral V particle with a proton as decay product, thus correctly distinguishing it as a baryon. Λ hyperon is the lightest particle and it can s[1]tay in contact with nucleons inside the nucleus and form lambda-hypernucleus. It has zero charge, zero isospin, the strangeness number -1 and the value of mass 1115.684 ± 0.006 MeV/c^2. Λ hyperon is composed of three quarks; (uds).

In today's nuclear physics, new nuclei or nuclear matter have been searched for along new axes such as isospin, spin and new flavor, namely strangeness. In the strangeness nuclear physics, new nuclei which has strangeness quantum number has been produced to study its structure and hence to gain information about nuclear force with strangeness. A hypernucleus consists of one or more hyperons bound to a nuclear core in addition to nucleon. The first hypernucleus was discovered in Warsaw in September 1952 by Marian Danysz and Jerzy Pniewski. The first hypernucleus was discovered in nuclear emulsion experiments as a fragment from the nuclear reaction by cosmic ray. A Λ-hypernucleus ${}^{A}_{\Lambda}X$ is a bound state of Z protons, (A-Z-1) neutrons and a Λ-hyperon.

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Interactions

In research paper, we use the phenomenological Λ-nucleus potential which has the Woods-Saxon form as in equation (1).

$$V_{\Lambda}(r) = \frac{V_s}{1 + \exp\left(\frac{r-R}{a}\right)} \qquad (1)$$

where, V_0=depth of the potential well

A=mass number of the core nucleus

V_s=strength of Woods-Saxon potential

R=radius of the core nucleus

a =the diffuseness parameter

Phenomenological Woods-Saxon Potential ForΛ-Core Nucleus

In this calculation, we considered that a Λ moves freely in an average potential well generated by the other nucleus. To study the Λsingle-particle energy level of ${}^{139}_{\Lambda}La$, the phenomenological Woods-Saxon potential energies is used which is described by the following equation (2),

$$\mathbf{V_{w.s} = -V_0\, \rho(r)} \qquad (2)$$

where, V_0 is the strength of Woods-Saxon potential and the nuclear density $\boldsymbol{\rho}(\mathbf{r}) = \frac{\mathbf{1}}{\mathbf{1+e^{r-R/a}}}$.

Woods-Saxon potential together with spin-orbit interaction which is described as follows:

$$\mathbf{V_{\ell.s} = V_{so}\left(\frac{\hbar}{m_{\pi}c}\right)^2 (\ell.s)\frac{1}{r}\frac{d\rho(r)}{dr}} \qquad (3)$$

Woods-Saxon potential including spin-orbit interaction is

$$\mathrm{V(r) = V_{w.s}(r) + V_{\ell.s}(r)}$$

Thus, the potential becomes;

$$\mathbf{V(r) = -V_o\,\rho(r) + V_{so}\left(\frac{\hbar}{m_{\pi}c}\right)^2 (\ell.s)\frac{1}{r}\frac{d\rho(r)}{dr}} \qquad (4)$$

where, R=nuclear radius=$r_0A^{1/3}$=$1.1A^{1/3}$ fm

r = radial distance from the centre

a = diffuseness parameter=0.6fm

V_0 = strength of Woods-Saxon potential

V_{so} = spin-orbit constant and

$\frac{\hbar}{m_{\pi}c}$ = Compton wavelength

The chosen parameters for the potential strength V_{so} and V_0 are 4MeV and 30 MeV.
The scalar product of LS coupling is

$$\boldsymbol{\ell.s} = \frac{1}{2}\left(\mathbf{j(j+1)} - \ell(\ell+1) - \frac{3}{4} \right) \tag{5}$$

If the potential is the phenomenological Woods-Saxon

For $j = \ell + \frac{1}{2}$ state, $\ell.s = \frac{1}{2}\ell$ and the potential can be expressed as the following equation,

$$\mathbf{V(r)} = \frac{-\mathbf{V_0}}{\mathbf{1 + e^{(r-R)/a}}} - \mathbf{V_{so}}\left(\frac{\hbar}{\mathbf{m_\pi c}}\right)^2 \left(\frac{1}{2}\ell\right)\left(\frac{1}{\mathbf{r}}\frac{\mathbf{e^{(r-R)/a}}}{\left(\mathbf{1+e^{(r-R)/a}}\right)^2}\frac{1}{\mathbf{a}}\right) \tag{6}$$

For $j = \ell + \frac{1}{2}$ state, $\ell.s = -\frac{1}{2}(\ell+1)$ and we can express the potential as

$$\mathbf{V(r)} = \frac{-\mathbf{V_0}}{\mathbf{1 + e^{(r-R)/a}}} + \mathbf{V_{so}}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{2}(\ell+1)\left(\frac{1}{\mathbf{r}}\frac{\mathbf{e^{(r-R)/a}}}{\left(\mathbf{1+e^{(r-R)/a}}\right)^2}\frac{1}{\mathbf{a}}\right) \tag{7}$$

Calculations

Numerov Method

The Schrodinger Radial Equation (SRE) is

$$\frac{\mathbf{d^2u(r)}}{\mathbf{dr^2}} + \frac{\mathbf{2\mu}}{\hbar^2}\left[\mathbf{E - V(r)} - \frac{\hbar^2}{\mathbf{2\mu}}\frac{\ell(\ell+1)}{\mathbf{r^2}}\right]\mathbf{u(r) = 0} \tag{8}$$

where E is total energy of the system, V is the potential energy of the system due to the forces acting between the two nucleons, μ is reduced mass and u(r)= r R_{nl} is the reduced radial wave function.

At the origin: $u(r) : u(r \to 0) \to r^{\ell+1}$

The asymptotic solution at $r \to \infty : u(r \to \infty) \to u(r) = e^{-\propto r^2}, \propto = \text{constant}$

The SRE can be written as $\frac{\mathbf{d^2u(r)}}{\mathbf{dr^2}} + \mathbf{k(r)u(r) = 0}$ (9)

$$k(r) = \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\right]$$ is kernel equation of the equation:

Here (setting $\ell = 0$), Schrodinger's Equation becomes $\frac{d^2u(r)}{dr^2} = u''(r) = -k(r)u(r)$

can be solved by Numerov Algorithm as follows:

Firstly, we split the r range into N points according to $r_n = r_{n-1} + h$ (where h is the step); then we write the wave function $u_n \equiv u(r_n) = u(r_{n-1} + h)$, and $k_n \equiv k(r_n) = k(r_{n-1} + h)$.

By using Taylor's series, we calculate the Forward Recursive Relation and Backward Recursive Relation, Forward Recursive Relation,

$$\mathbf{u_n} = \frac{2\left[1-\frac{5h^2}{6}k_{n-1})\right]u_{n-1} - \left[1+\frac{h^2}{12}k_{n-2}\right]u_{n-2}}{\left[1+\frac{h^2}{12}k_n\right]} \qquad (10)$$

Backward Recursive Relation,

$$\mathbf{u_{n-1}} = \frac{2\left[1-\frac{5h^2}{6}k_n\right]u_n - \left[1+\frac{h^2}{12}k_{n+1}\right]u_{n+1}}{\left[1+\frac{h^2}{12}k_{n-1}\right]} \qquad (11)$$

Therefore, when we calculate our wave function using the backward-forward recursive technique, we should note that the recursive formulas imply having knowledge of two initial values for each direction. Since both $u_{out}(r)$ and $u_{in}(r)$ satisfy a homogeneous equation, their normalization can always be chosen so that they are set to be equal at the r_c point. An energy eigen value is then signaled by the equality of derivatives at this point.

At the matching point the eigen functions $u_{out}(r)$ and $u_{in}(r)$ and first derivatives $u_{out}(r)$ and $u_{in}(r)$ must all satisfy the continuity conditions:

$$\left.\begin{array}{l} (u_{out})_{r_c} = (u_{in})_{r_c} \\ (u'_{out})_{r_c} = (u'_{in})_{r_c} \end{array}\right\}$$

thus, we can write the corresponding condition for the logarithmic derivative at r_c as

$$\left[\frac{\mathbf{u'_{out}}}{\mathbf{u_{out}}}\right]_{\mathbf{r_c}} = \left[\frac{\mathbf{u'_{in}}}{\mathbf{u_{in}}}\right]_{\mathbf{r_c}} \qquad (12)$$

and then we define a G(E) function at whose value zeros correspond to the energy eigenvalues as

$$\mathbf{G(E)} = \left[\frac{\mathbf{u'_{out}}}{\mathbf{u_{out}}}\right]_{\mathbf{r_c}} - \left[\frac{\mathbf{u'_{in}}}{\mathbf{u_{in}}}\right]_{\mathbf{r_c}} \qquad (13)$$

Therefore, we proceed numerically in the following way: we set a trial energy range splitting this E range into N points, corresponding to where is the energy step. For each we calculate their eigenfunctions and at the point; and we build the G(E) function here, looking for a change of sign in it (which implies a zero cross). Once we find it, we perform a fine turning closing the energy range until the required tolerance.

Root-mean square distance

Root-mean-square distance means the maximum probability distance between Lanthanum core nucleus and lambda particle. The root-mean square distance is evaluated by the following equation with the numerical calculated results of wave functions.

$$\left\langle \sqrt{\bar{\mathbf{r}}_{\mathbf{rms}}} \right\rangle^2 = \int \mathbf{\Psi^* r^2 \Psi dr} \qquad (14)$$

Results and Discussion

The Λ single-particle energies of ${}^{139}_{\Lambda}La$ and corresponding wave functions have been investigated by solving Numerically Schrodinger radial equation with Numerov Method. Numerov Method can be evaluated both energy eigen values and wave functions simultaneously. This method can be calculated both bound state problems and scattering problems. Bound state energies give negative values and Scattering gives positive values. In this paper, about the bound state energies have been calculated. And also calculated the radial wave functions and corresponding root mean square distances. Lambda single particle energy levels are summarized in Table (1).The calculated results of Binding energy with W-S potential without L-S term are shown in Table (2). The single-particle energy levels of ${}^{139}_{\Lambda}La$ is are $1s_{1/2}$, $1p_{1/2}$, $1p_{3/2}$, $1d_{3/2}$, $1d_{5/2}$, $1f_{5/2}$, $1f_{7/2}$, $1g_{9/2}$, $2s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$. The energy level diagrams for these Λ hypernuclei are in Figure (5). The calculated results by using Woods-Saxon potential including spin-orbit interaction are in good agreement with H.BANDO results and quite different from experimental results. To agree with experimental data, the calculations need to consider the additional potential such as spin-spin interaction, paring effect etc.

Table (1) BANDO calculated results, calculated results and experimental results with W-S potential including L-S term.

States	H. BANDO Calculated Results	Calculated Results (MeV)	Experimenal results (MeV)	Calculated RMS (fm)
$1s_{1/2}$	-24.7	-24.66	-24.5 ± 0.6	1.6064
$1p_{1/2}$		-19.45	-20.4 ± 0.6	1.911
$1p_{3/2}$	-19.7	-19.73		1.9277
$1d_{3/2}$		-13.39	-14.3 ± 0.6	2.1484
1d5/2	-14.1	-14.02		2.1745
1f5/2		-6.635	-8.0 ± 0.6	2.3703
$1f_{7/2}$	-7.8	-7.75		2.3936
$1g_{9/2}$	-1.2	-1.087		2.6239
2s1/2		-12.06		2.0649
2p1/2		-4.95		2.4016
2p3/2	-5.4	-5.34		2.3955

Table (2) The calculated results of Binding energy with W-S potential without L-S term.

States	1s	1p	2p	1d	1f
Calculated Results (MeV)	-24.658	-19.638	-5.209	-13.77	-7.265
RMS (fm)	1.6066	1.9219	2.3975	2.1634	2.3844

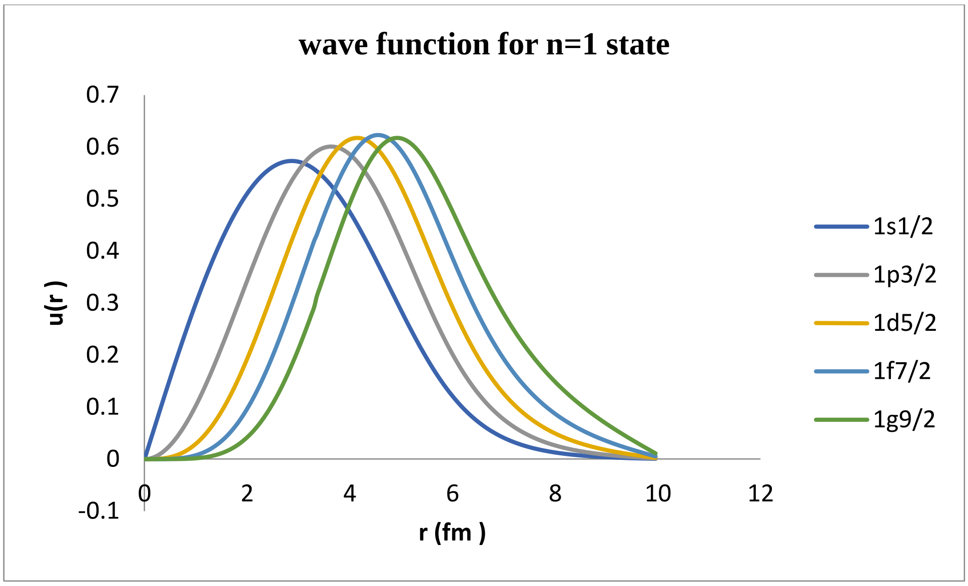

Figure 1 Behaviour of wave function of Lambda in Woods-Sexon potential well for n = 1 state ($^{139}_{\Lambda}La$)

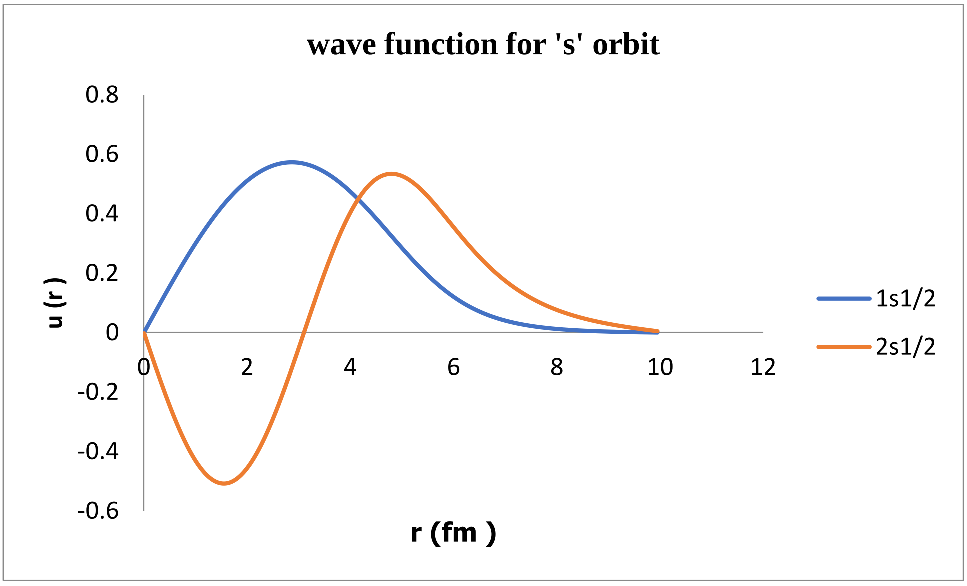

Figure 2 Behaviour of wave function of Lambda in Woods-Sexon potential well for $1s_{1/2}$ state and $2s_{1/2}$ state ($^{139}_{\Lambda}La$)

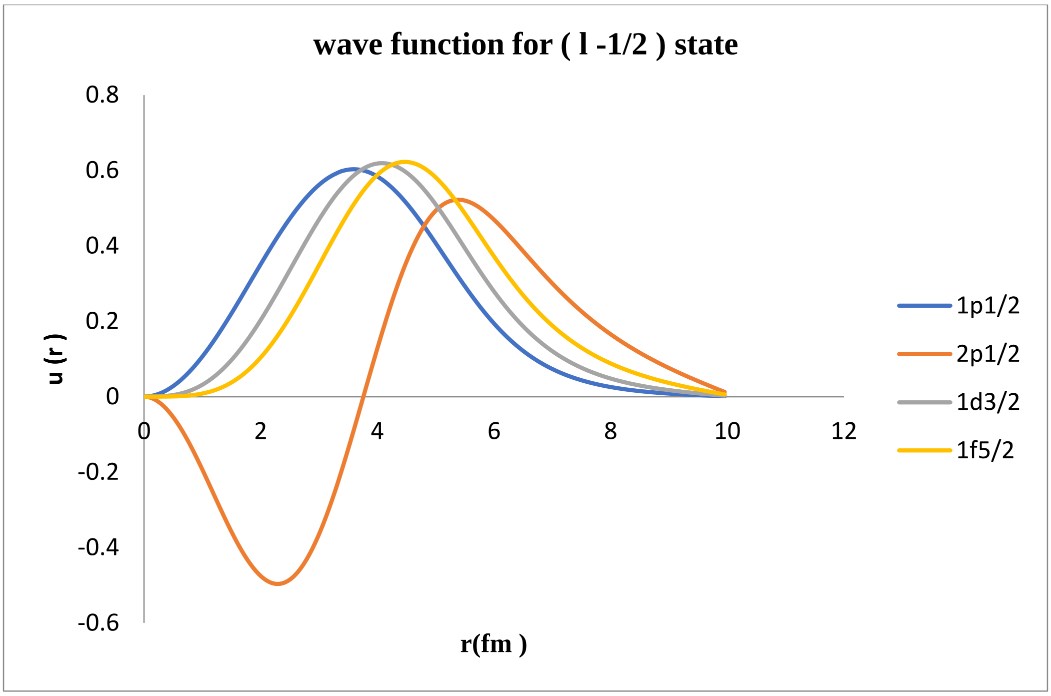

Figure 3 Behaviour of wave function of Lambda in Woods-Sexon potential well for $\left(\ell - \frac{1}{2}\right)$ state in (${}^{139}_{\Lambda}La$)

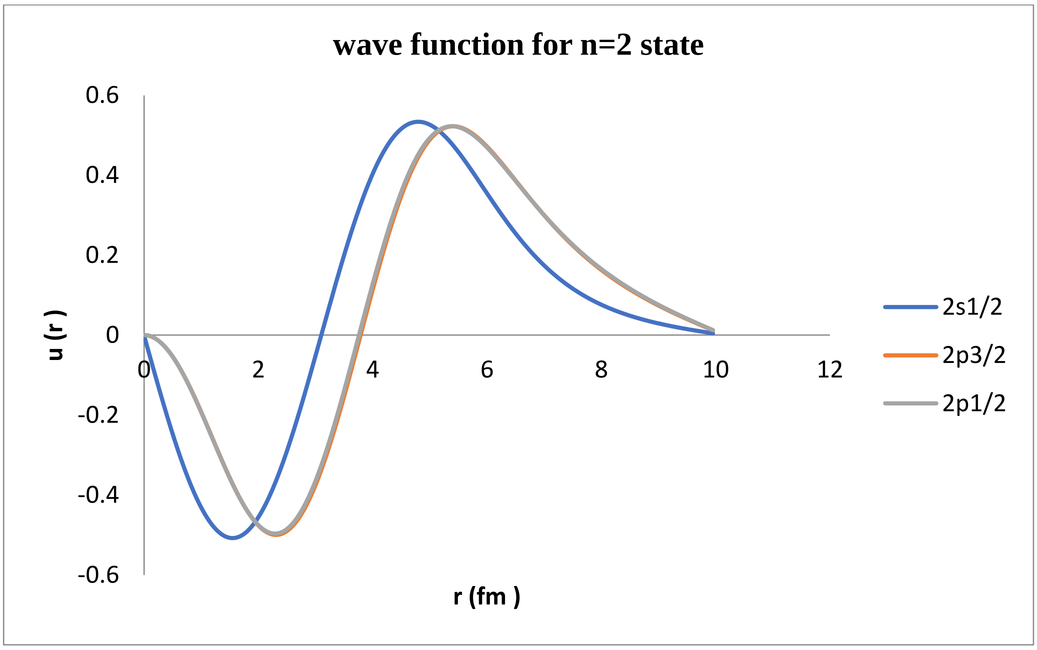

Figure 4 Behaviour of wave function of Lambda in Woods-Sexon potential well for $2s_{1/2}$ state, $2p_{3/2}$ state and $2p_{1/2}$ state (${}^{139}_{\Lambda}La$)

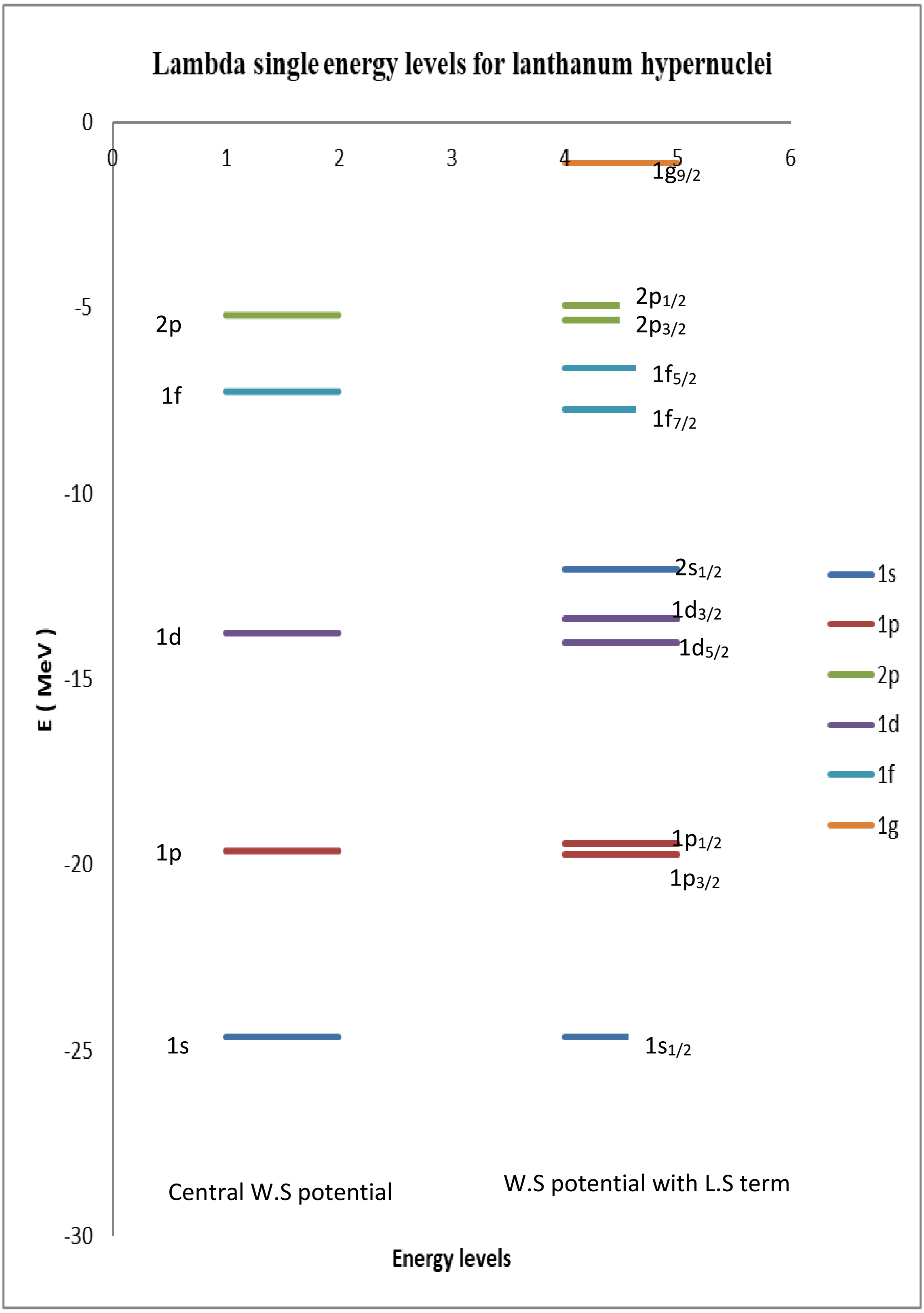

Figure 5 Energy level diagram of $^{139}_{\Lambda}La$ for Woods-Saxon potential with central term and Woods-Saxon potential including spin orbit term.

Conclusion

The Λ hyperon does not suffer from Pauli blocking by the other nucleons, it can penetrate into the nuclear interior and form deeply bound hypernuclear states. There are no nodes for principal quantum number n = 1, one node for n = 2. The wave function is shifted to outer region for higher orbital angular momentum. The higher the orbital angular momentum, the higher the spacing between $(\ell + \frac{1}{2})$ and $(\ell - \frac{1}{2})$. According to calculated results, there is no nuclear bound state above $1g_{9/2}$ state for ${}^{139}_{\Lambda}\mathrm{La}$ system. After the research, the analytical and numerical calculation about the Numerov method have been known. And also, the behavior of wave functions, energy levels and the corresponding root mean square distances have been known.

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