CONSTRUCTIONOF $K^{-}p$ SEPARABLE POTENTIALFROM $\Lambda(1405)$ PARAMETERS

Khin Soe Moe[1], Nay Min Oo[2] and Khin Swe Myint[3]

Abstract

$K^{-}p$ separable potential which is reproduced from the $\Lambda(1405)$ energy and level width, 27MeV and 40MeVhas been constructed. The Schrodinger equation has been solved by using our constructed separable potential. And then, unknown data of potential strength parameter, $\overline{V_0}$, and potential range parameter, b, have been calculated by comparing the binding energy and level width of $\Lambda(1405)$.The constructed separable potential model is $V(\vec{r},\vec{r}\,') = 1.34\,\frac{1}{r}\,\frac{1}{r'}\,e^{-\left(\frac{r+r'}{1.26}\right)}$.

Keywords: single $K^{-}p$ separable potential, potential strength parameter, potential rangeparameter

Introduction

$\overline{K}N$ interaction can be theoretically constructed from the experimental data of $\Lambda(1405)$ quasi bound state and $\overline{K}N$ scattering parameters. It is accepted that $\Lambda(1405)$ is quasi bound state with isospin I = 0 which is composed of K^{-} and proton. The strange resonance $\Lambda(1405)$ has $J^{\pi} = \frac{1}{2}^{+}$ and is found in s-wave $\Sigma\pi$ scattering(Akaishi Y and Yamazaki T.).

It is very important to understand the kaon-nucleon interaction and its modification due to nuclear medium, which leads to the kaon-nucleus optical potential, for the discussion of deeply bound kaonic states. In particular, the understanding of the $\Lambda(1405)$ is essential in order to study the fate of the kaons in the nucleus. We have solved the Schrodinger equation by using our constructed separable potential. And then, unknown data of potential strength parameter, $\overline{V_0}$, and potential range parameter, b, have been calculated by comparing the binding energy and level width of $\Lambda(1405)$.

Theoretical Review on Kaonic Nuclei

The search for deeply bound hadronic states has a long history. The hadron which attracted much attention recently is kaon. The exotic nuclear systems involving a $\overline{K}\left(K^{-} \text{ or } \overline{K^{0}}\right)$ as a constituent have been investigated theoretically based on phenomenologically constructed $\overline{K}N$ interaction, which reproduces low-energy $\overline{K}N$ scattering data, kaonic hydrogen atom data and the binding energy and decay width of $\Lambda 1405$.

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In recent years, there have been important developments in the studies of kaonic nuclear states, which are kaon-nucleus systems bound by the strong interaction inside the nucleus. The very interesting feature of the kaon-nucleus bound systems is based on the fact that the properties of kaons in nuclei are strongly influenced by the change undergone by $\Lambda(1405)$ in nuclear medium, because $\Lambda(1405)$ is a resonance state just below the kaon-nucleon threshold. There are studies of kaonic atoms carried out by modifying the properties of $\Lambda(1405)$ in nuclear medium.

Yamazaki and Akaishi theoretically investigated the possible existence of deeply bound kaonic nuclei. The nuclear ground states of K^- in ^{3}He, ^{4}He and ^{9}Be are theoretically predicted to be discrete states with binding energies of 108 MeV, 86 MeV and 113 MeV and widths of 20 MeV, 34 MeV and 38 MeV respectively. Energy level diagram of the $K^- + {}^3$He system is shown in Fig 1.

For three-body K^-pp system, Akaishi *et al.* have predicted that K^-pp system is bound with a binding energy (E_K = 48 MeV) and a width (Γ_K = 61 MeV). The average distance between two protons in this three-body system is found to be 1.9 fm(Akaishi Y and Yamazaki T.).

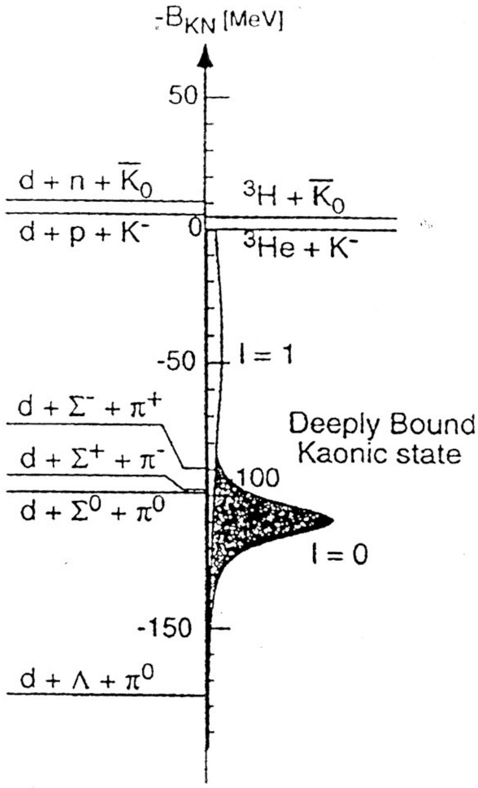

Figure 1 Energy level diagram of the $K^- + {}^3$He system and the energies of relevant decay channels.

$\bar{K}$N Potential

Kaonic atoms have been phenomenologically studied, and the large strength of the potential assumed at a time raised hopes that deeply bound states (bound by 50_200 MeV) could exist. However, the imaginary part of the optical potential is large and therefore, the widths of the deeply bound kaonic states come out to be too large to be detected as distinguished states. Hence, one should rely on the microscopic derivation of the kaon optical potential to find a mechanism for small widths of deeply bound kaonic states, if they exist. The microscopic derivation of the optical potential for kaonic atoms is related strongly to the properties of the $\Lambda(1405)$ state, which is located just below the kaon-proton threshold. There have been many studies on the theoretical derivation of kaon nucleus optical potential.

$\bar{K}N$ interaction can be theoretically constructed from the experimental data of $\Lambda(1405)$ quasi bound state and scattering parameters. It is accepted that $\Lambda(1405)$ is a quasi bound state with isospin I = 0 which is composed of K^- and proton.

Akaishi and Yamazaki consider the $\Lambda(1405)$ state as the bound state of the kaon and proton and its width is caused by the coupling to the π-Σ channel.

Mathematical Formulation

A separable potential with a Yukawa type form factor can be expressed as follows.

$$V(\vec{r},\vec{r}\,') = g(r)\,Y_{lm}\,\frac{\bar{V}_0}{b^3}\,g(r')\,Y_{l'm'}$$

where,

$\bar{V}_0$ = potential strength parameter

b = potential range parameter

g(r) = form factor

We will construct the K^-p optical potential from this separable potential.

Schrodinger equation of two-body system in single channel is expressed as:

$$H|\psi\rangle = E|\psi\rangle \tag{1}$$

$$\langle\psi|T+V|\psi\rangle = E\langle\psi|\psi\rangle \tag{2}$$

$$\iint d\vec{r}\,d\vec{r}\,'\langle\psi|\vec{r}\rangle\langle\vec{r}|T|\vec{r}\,'\rangle\langle\vec{r}\,'|\psi\rangle + \iint d\vec{r}\,d\vec{r}\,'\langle\psi|\vec{r}\rangle\langle\vec{r}|V|\vec{r}\,'\rangle\langle\vec{r}\,'|\psi\rangle = E\int d\vec{r}\langle\psi|\vec{r}\rangle\langle\vec{r}|\psi\rangle \tag{3}$$

$$\begin{aligned}\iint d\vec{r}\,d\vec{r}\,'\psi^*(\vec{r})\left(-\frac{\hbar^2\vec{\nabla}^2}{2\mu}\right)\delta(\vec{r}-\vec{r}\,')\psi + \iint\iint d\vec{r}\,d\vec{r}\,'\psi^*(\vec{r})g(\vec{r})\,Y_{00}\,\frac{\vec{V}_0}{b^3}\,g(\vec{r}\,')\,Y_{00}\,\psi(\vec{r}\,') \\ = E\int d\vec{r}\,\psi^*(\vec{r})\psi(\vec{r})\end{aligned} \tag{4}$$

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\psi(\vec{r}) + g(\vec{r})Y_{00}\frac{\vec{V}_0}{b^3}\int g(\vec{r}\,')Y_{00}\,\psi(\vec{r}\,')d\vec{r}\,' = \frac{\hbar^2}{2\mu}k^2\,\psi(\vec{r}) \tag{5}$$

where

$$E = \frac{\hbar^2k^2}{2\mu} \quad ; \quad k = \text{wave vector}$$

$$\psi(\vec{r}) = \frac{u(r)}{r} Y_{00} \ ; \quad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

We will rewrite the equation (5) as follows:

$$-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2}\frac{u(r)}{r}Y_{00} + g(\vec{r})Y_{00}\frac{\vec{V}_0}{b^3}\int g(\vec{r}')Y_{00}\, d\vec{r}' = \frac{\hbar^2}{2\mu}k^2\frac{u(r)}{r}Y_{00} \tag{6}$$

$$-\frac{d^2}{dr^2}u(r) + G_0\, r\, g(r)\int g(r')\frac{u(r')}{r'}\frac{1}{4\pi}dr'^{\,r'} = k^2 u(r) \tag{7}$$

Where, $G_0 = \frac{2\mu}{\hbar^2}\frac{\vec{V}_0}{b^3}$

$$-\frac{d^2}{dr^2}u(r) + G_0\, r\, g(r)\int g(r')u(r')r'\, dr' = k^2 u(r) \tag{8}$$

We let $G_0\int_0^{\infty} g(r')u(r')r'\, dr'$ in equation (8) as G to simplify our calculation.

Then equation (8) is written as:

$$-\frac{d^2}{dr^2}u(r) + r\, g(r)G = k^2 u(r) \tag{9}$$

Since wave function u(r) is the total wave function of inhomogeneous wave function $u_0(r)$ and homogeneous wave function $u_1(r)$, the equation (9) can be written as:

$$-\frac{d^2}{dr^2}u_0(r) - \frac{d^2}{dr^2}u_1(r) + r\, g(r)G = k^2 u_0(r) + k^2 u_1(r) \tag{10}$$

Equation (10) can be separated to inhomogeneous equation and homogeneous equation.

Inhomogeneous equation is

$$-\frac{d^2}{dr^2}u_0(r) + r\, g(r)G = k^2 u_0(r) \tag{11}$$

Homogeneous equation is

$$-\frac{d^2}{dr^2}u_1(r) = k^2 u_1(r) \tag{12}$$

We will solve equation (11) as follows:

$$\frac{d^2}{dr^2}u_0(r) - r\, g(r)G = -k^2 u_0(r) \tag{13}$$

Then

$$u_0(r) = A\, e^{-r/b} \tag{14}$$

where, $A = \dfrac{Gb^3}{1+k^2b^2}$

Then, by solving equation (12), we get

$$u_1(r) = a \sin (kr + \delta) \tag{15}$$

Since $u(r) = u_0(r) + u_1(r)$,

$$u(r) = A\, e^{-r/b} + a \sin(kr+\delta) \tag{16}$$

For boundary condition, $u(0) = 0$

$$A = -a \sin \delta \tag{17}$$

Therefore, equation (16) can be expressed as follows:

$$u(r) = A\, e^{-r/b} - Ae^{ikr} \tag{18}$$

Since

$$G = G_0 \int_0^{\infty} g(r) u(r)\, r dr,$$

$$\frac{G}{G_0} = \int_0^{\infty} \frac{b}{r}\, e^{-\frac{r}{b}} \{Ae^{-\frac{r}{b}} - A\, e^{-ikr}\}\, r dr \tag{19}$$

Then,

$$\frac{G}{G_0} = -\frac{1}{2} Ab^2 \left(\frac{1+ikb}{1-ikb}\right) \tag{20}$$

Since $A = \dfrac{Gb^3}{1+k^2b^2}$

Since $A = -a \sin\delta$, then

$$-a \sin\delta = \frac{Gb^3}{1+k^2b^2} \tag{21}$$

$$G = -a \sin\delta\, \frac{1+k^2b^2}{b^3}$$

When equation (21) is divided by G_0, equation (22) will be as follows.

$$\frac{G}{G_0} = -a \sin\delta\, \frac{1+k^2b^2}{G_0 b^3} \tag{22}$$

From equation (20) and equation (22), we get equation (23)

$$-\frac{1}{2} Ab^2 \left(\frac{1+ikb}{1-ikb}\right) = A \frac{(1+ikb)(1-ikb)}{G_0 b^3} \tag{23}$$

Then. $(1-ikb)^2 = -s \text{ where } s = \frac{1}{2} G_0 b^5$ (24)

Then, $k = -i\frac{1}{b} \pm \frac{1}{b}\sqrt{s}$ $\frac{\hbar^2}{2\mu}\frac{1}{b^2}$

In calculation, we let as B_0.

Our calculated resonance energy can be written as follows :

$$E_{Res} = B_0(s-1) - i(2B_0\sqrt{s}) \quad (25)$$

Where, $B_0 = \frac{\hbar^2}{2\mu}\frac{1}{b^2}$

Experimental value of resonance energy for $\Lambda(1405)$ is

$$E_{Res} = -27 - i40 \text{ MeV} \quad (26)$$

By comparing our calculated results with the experimental data of $\Lambda(1405)$.

$$-27 - i40 = B_0(s-1) - i(2B_0\sqrt{s}) \quad (27)$$

$$-27 - i\,40 = \frac{\hbar^2}{2\mu}\frac{1}{b^2}\frac{\mu}{\hbar^2}\overline{V}_0\, b^2 - \frac{\hbar^2}{2\mu}\frac{1}{b^2} - i\left(\frac{\hbar^2}{\mu}\frac{1}{b^2}\sqrt{\frac{2\mu\overline{V}_0\, b^2}{2\hbar^2}}\right) \quad (28)$$

By comparing real value and imaginary value of the above equation, we get equation (29) and equation (30).

$$-27 = \frac{\overline{V}_0}{2} - \frac{\hbar^2}{2\mu\, b^2} \quad (29)$$

$$40 = \sqrt{\frac{2\hbar^2\overline{V}_0}{2\mu\, b^2}} \quad (30)$$

Then we get the value of potential strength parameter, $\overline{V}_0$.

$\overline{V}_0$ = 21.26 MeV (or) –75.26 MeV

For $\overline{V}_0$ = 21.26 MeV, b = 1.26 fm (or) –1.26 fm

For $\overline{V}_0$ = –75.26 MeV, b = i2.38 fm (or) –i2.38 fm

For resonance state, it must be $G_0 > 0$ and $s > 0$.

Finally, our suitable results are

$\overline{V}_0$ = 21.26 MeV and b = 1.26 fm

Yukawa type non-local potential is

$$V(\vec{r}, \vec{r}) = g(r) Y_{1m} \frac{\overline{V}_0}{b^3} g(r') Y_{1'm'}$$

Then, we get our constructed separable potential by using calculated results of potential strength parameter, $\overline{V}_0$, and potential range parameter, b.

$$V(\vec{r},\vec{r}') = 1.34\frac{1}{r}\frac{1}{r'}e^{-\left(\frac{r+r'}{1.26}\right)}. \tag{31}$$

Results and Discussion

To analyze the existence of the kaonic nucleus, we must have a reliable kaon nucleon interaction in the nuclear medium. For this requirement, we construct the $\overline{K}N$ separable potential. It is Yukawa type separable potential and is constructed analytically and numerically. In our constructed separable potential model, unknown data of potential strength parameter, $\overline{V}_0$, and potential range parameter, b, have been computed.

By comparing our calculated resonance energy with the energy of $\Lambda(1405)$, we calculate the unknown data of $\overline{V}_0$ and b. In our calculation, the value of potential strength parameter, $\overline{V}_0$, is 21.26 MeV. That of potential range parameter, b, is 1.26 fm. Then, the $\overline{K}N$ Yukawa type separable potential, which reproduces the experimental values of $\Lambda(1405)$, is constructed as

$$V(\vec{r},\vec{r}) = 1.34\frac{1}{r}\frac{1}{r}e^{(\frac{r+r}{1.26})}.$$

This potential is non-local potential that can separate the local form. So, it is also called separable potential.

Computation difficulty will be encountered in solving the kaon nucleus system with the separable potential in coordinate space. This difficulty can be overcome by constructing a local potential which is equivalent to the non- local separable potential.

Conclusion

By using our constructed separable potential, we can calculate the binding energy of kaonic nuclei and can do the structure analysis of this system.

Scattering parameters such as cross section, phase shift, scattering length, effective range, etc. can be computed by using our constructed potential. If the calculated results will be confirmed experimentally, we can say that our constructed separable potential is good and our present work is a timely research in this frontier of physics.

If anyone interest in research, he can continue to calculate the scattering parameters by using our constructed separable potential and compare the experimental results.

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