COMPARISON BETWEEN SINGLEPARTICLE ENERGY STATES OF $^{12}_{\Lambda}C$ AND $^{12}_{6}C$

Cho Cho San[1], Khin Swe Myint[2]

Abstract

Nucleon single-particle energy states in ordinary nuclei and Λ single particle energy states in Λ-hypernuclei are compared to study the role of Λ particle in nuclear medium. So, we examined the single particle energy state in Λ-hypernuclei $^{12}_{\Lambda}C$ and ordinary nuclei $^{12}_{6}C$. In our calculation, in order to get the single particle energy levels of $^{12}_{\Lambda}C$ and $^{12}_{6}C$, we used the phenomenological Woods-Saxon Λ-core nucleus potential and nucleon-nucleus potential including spin-orbit term. Gaussian basis wave function is used in our consideration systems.

Key words: Λ single particle level, nucleon single particle level

Introduction

The study of the response of many-body system to a hyperon gives insight into the information of baryon-baryon interactions. The valuable information on the Λ-N interaction has an impact upon understanding of baryon-baryon interaction within the framework of SU (3) flavor symmetry. Since a bound Λ hyperon inside the nucleus can only decay via weak interaction, life time of hypernuclei is long enough to be observed and the binding data of hypernuclei provide a unique opportunity to know more about the Λ-nuclear interaction.(Hasegawa, T.,et al.,(1995))

Also the determination of the Λ spin-orbit interaction is very important to totally understand the nature of spin-orbit interaction of Λ and ordinary nucleus. It is known that the Λ-nucleus spin- orbit splitting is at least an order of magnitude smaller than that of the nucleon-nucleus interaction. Spin and orbit refer to the attributes of a single nucleon moving in the assumed potential. Such a term is found in atomic physics and it is due to the magnetic interaction of the magnetic dipole moment of the electron spin and the magnetic field experienced by the electron in its rest frame as moves through the Coulomb field of the nucleus. However, nuclear spin-orbit interaction

[1.] Dr, Lecturer, Department of Physics, Mandalay University of Distance Education, Mandalay, Myanmar.

[2.] Department of Physics, Mandalay University, Mandalay, Myanmar.

cannot be magnetic in origin as it is notstrong enough to explain the observed spin-orbit splitting. We have calculated Λ single particle energy ${}^{12}_{\Lambda}C$ and nucleon single particle energy levels of ${}^{12}_{6}C$ by using Λ-core nucleus and nucleon-nucleus phenomenological Woods-Saxon potentials including spin orbit term.

Glue-Like Role of Λ- Hypernuclei

The structure is modified when a hyperon, a Λ particle, is injected into the nucleus. There is no Pauli principle acting between the Λ and the nucleons in the nucleus. Therefore, the Λ particle can reside deep inside, and attract the surrounding nucleons towards the interior of the nucleus. Then, the Λ particle plays a 'glue like role' to produce a dynamical contraction of the core nucleus. Nuclei are compressed by the injection of a Λ particle such that the matter radius in some excited states is shrunk by as much as 30% due to the addition of a Λ particle.

Λ can occupy whatever single particle state, the ground state of the hypernucleus always corresponding to the hyperon in the 1s level. It is then clear that a Λ particle is a good probe of the inner part of nuclei. Actually, the Pauli principle is active on the u and d quarks of nucleons and Λ when they are very close to each other. With the exception of hypernuclei of the s-shell, the depth of the Λ-nucleus mean field is of about 30 MeV, it is less attractive than the one typical for a nucleon (50-55 MeV). This characteristic reflects the smaller range and the weakness of the ΛN interaction at intermediate distances with respect to the NN one. It is possible to reproduce the experimental single particle Λ levels using Woods-Saxon wells with the depth and appropriate radii. For s-shell hypernuclei the Λ single particle potential displays a repulsive soft core at short distance. A measure of this effect is given by the rms radii for a nucleon and a Λ in these hypernuclei: the hyperon rms radius is larger than the one for a nucleon. In order to study the Λ single-particle energy levels, we determine the Λ-nucleon spin-orbit splitting.

The Lambda Nucleon Spin-Orbit Splitting

The properties of baryon many-body systems, which contain not only nucleons but also hyperons with strangeness, link closely to the underlying hyperon-nucleon interaction. The determination of the Λ spin orbit interaction is very important to totally understand the nature of spin orbit interaction of Λ and ordinary nucleus. Several features of the Λ single-particle properties in the nucleus, being essentially different from those of the nucleon, have clearly emerged from these efforts. It is well accepted nowadays that the depth of the Λ-nucleus potential is around -30 MeV, which is 20MeV less attractive than the corresponding nucleon-nucleus one. The spin orbit splitting of single particle levels in Λ hypernuclei were found to be much smaller than their nucleonic counterparts, typically more than one order of magnitude. Such effect could originate from the weak tensor component of the Λ-N interaction. Moreover, the Λ, contrary to the nucleon, maintains its single particle character even for states well below the Fermi surface indicating a weaker interaction with other nucleons.

From the experimental motivation, the binding energy of ${}^{12}_{\Lambda}\mathrm{C}$ which is observed from the emulsion experiment is $-10.08 \pm 0.18\,\mathrm{MeV}$. Moreover the spectroscopic study of ${}^{12}_{\Lambda}\mathrm{C}$ by the (π^+, K^+) reaction was reported by T. Haseagawa and his collaboration.

Hypernuclei structure calculations with core-excited states will be important in future analysis.

The Nucleon Nucleon Spin-orbit Splitting

Spin orbit splitting of nuclei is one of the main factors, which determine nuclear structure in nuclei both near and far from the closed shells. Spin and orbit refer to the attributes of a single nucleon moving in an averaged potential well. Spin-orbit splitting in atoms is due to the magnetic interaction of the magnetic dipole moment of the electron spin and the magnetic field experienced by the electron in its rest frame as it moves through the Coulomb field of the nucleus. However, the dynamical origin of the strong nuclear spin orbit force has not been fully resolved even up to date. The analogy with the spin orbit interaction in atomic physics gave the hint that it could be a relativistic effect. This idea has led to the construction of the relativistic scalar-vector mean field models for nuclear structure calculation. In these models the nucleus is described as a collection of independent Dirac-particles moving in self-consistently generated scalar and vector mean-fields. The footprints of relativity become visible through the large nuclear spin orbit coupling which emerges in that framework naturally from the interplay of two strong and counteracting (scalar and vector) mean-fields. The corresponding spin orbit term comes out proportional to the coherent sum of the very large scalar and vector mean-fields. In this sense, the relativistic mean-field model gives a simple and natural explanation of the basic features of the nuclear shell model potential.

The Λ-Core Nucleus Interaction

Single particle potential of hyperons is fundamental quantities in hypernuclei many-body systems, which are closely related to the properties of hyperon nucleon interaction. To investigate single particle energy levels of ${}^{12}_{\Lambda}C$ and ${}^{12}_{6}C$, we employ the following potential, phenomenological Woods-Saxon Λ-core nucleus potential.

Phenomenological Woods-Saxon Λ-core Nucleus Potential

$$v\,(r) = -v_O f(r) + v_{SO}\left(\frac{\hbar}{m_\pi c}\right)^2 (\vec{\ell}.\vec{s})\frac{1}{r}\frac{df(r)}{dr} \qquad (1)$$

$$f(r) = \frac{1}{1 + e^{\left(\frac{r-R}{a}\right)}}$$

where, v_0 = strength of the Woods-Saxon potential

v_{so}= spin-orbit constant

$\frac{\hbar}{m_\pi c}$ = Compton wavelength

a = diffuseness parameter

R = nuclear radius

The chosen parameters for nucleon are

$$v_0^N = 50 \text{ MeV}, v_{SO}^N = 14 \text{ MeV}, \; R = 1.25 \; A^{1/3} \text{ fm}, \; a = 0.53 \text{ fm} \tag{2}$$

The chosen parameters for lambda are

$$v_0^\Lambda = 30 \text{ MeV}, \; v_{SO}^\Lambda = 4 \text{ MeV}, \; R = 1.1(A-1)^{1/3} \text{ fm}, \; a = 0.6 \text{ fm} \tag{3}$$

The scalar product of $\vec{\ell}$ and $\vec{s}$ is

$$\vec{\ell}.\vec{s} = \frac{1}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right]\hbar^2 \tag{4}$$

Mathematical Formulation

Schrödinger equation for two-body bound system is

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2}{2\mu}\;\frac{\ell(\ell+1)}{r^2} + V(r)\right\}u(r) = E\,u(r) \tag{5}$$

The Gaussian form for wave function is

$$u(r) = r^{\ell+1}\sum_{j=1}^{N_b} c_j\, e^{-\left(\frac{r}{b_j}\right)^2} \tag{6}$$

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2}{2\mu}\;\frac{\ell(\ell+1)}{r^2} - v_o f(r) + v_{SO}\left(\frac{\hbar}{m_\pi c}\right)^2\left(\vec{\ell}.\vec{s}\right)\frac{1}{r}\;\frac{df(r)}{dr}\right\}$$

$$r^{\ell+1}\sum_{j=1}^{N_b} c_j\, e^{-\left(\frac{r}{b_j}\right)^2} = E\;\, r^{\ell+1}\sum_{j=1}^{N_b} c_j\, e^{-\left(\frac{r}{b_j}\right)^2} \tag{7}$$

By solving equation (7) we obtained the following norm matriterm, the kinetic energy term, the centrifugal term and the potential term.

The norm matrix term is

$$N_{ij}^{\ell} = \frac{(2\ell+1)!!\sqrt{\pi}}{2^{\ell+2}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{3}{2}}} \qquad (8)$$

The kinetic energy term is

$$T_{ij}^{\ell} = -\frac{\hbar^2}{2\mu}\left[\begin{array}{l}\frac{4}{b_j^4}\left(\frac{(2\ell+3)!!\sqrt{\pi}}{2^{\ell+3}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{5}{2}}}\right)-\frac{4\ell+6}{b_j^2}\left(\frac{(2\ell+1)!!\sqrt{\pi}}{2^{\ell+2}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{3}{2}}}\right)+ \\ \ell(\ell+1)\left(\frac{(2\ell-1)!!\sqrt{\pi}}{2^{\ell+1}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{1}{2}}}\right)\end{array}\right] \qquad (9)$$

The centrifugal potential term is

$$F_{ij}^{\ell} = \frac{\hbar^2}{2\mu}\ell(\ell+1)\left(\frac{(2\ell-1)!!\sqrt{\pi}}{2^{\ell+1}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{1}{2}}}\right) \qquad (10)$$

The scalar product of $\vec{\ell}$ and $\vec{s}$ is

$$\vec{\ell}.\vec{s} = \frac{1}{2}\left[j(j+1)-\ell(\ell+1)-s(s+1)\right]\hbar^2 \qquad (11)$$

For $j=\ell+\frac{1}{2}$ state, the potential term is

$$v_{ij}^{\ell} = \int r^{2\ell+2} e^{-\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)r^2} \left[\frac{-v_o}{1+e^{\frac{r-R}{a}}} + v_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \left(\frac{1}{2}\ell\right) \left(-\frac{1}{r} \frac{e^{\frac{r-R}{a}}}{\left(1+e^{\frac{r-R}{a}}\right)^2} \frac{1}{a} \right) \right] dr \qquad (12)$$

For $j = \ell - \frac{1}{2}$ state, the potential term is,

$$v_{ij}^{\ell} = \int r^{2\ell+2} e^{-\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)r^2} \left[\frac{-v_o}{1+e^{\frac{r-R}{a}}} + v_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \left(\frac{-(\ell+1)}{2}\right) \left(-\frac{1}{r} \frac{e^{\frac{r-R}{a}}}{\left(1+e^{\frac{r-R}{a}}\right)^2} \frac{1}{a} \right) \right] dr \qquad (13)$$

The Hamiltonian is expressed by summing of kinetic energy term, centrifugal term and potential term as follows:

$$H_{ij}^{\ell} = T_{ij}^{\ell} + F_{ij}^{\ell} + V_{ij}^{\ell} \qquad (14)$$

$$\sum_j H_{ij}^{\ell} c_j = E \sum_j N_{ij}^{\ell} c_j \qquad (15)$$

By rewriting equation (15) in terms of matrix form as follows:

$$[H][c] = E[N][c] \qquad (16)$$

The above equation (16) is numerically solved by using power inverse iteration method. Then we get the single particle energy levels ${}^{12}_{6}C$ and we compare them with the Λ-hypernuclei ${}^{12}_{\Lambda}C$.

Results and Discussion

We have calculated the single-particle energy levels for Λ-hypernuclei ${}^{12}_{\Lambda}C$ and ordinary nuclei ${}^{12}_{6}C$ by solving two-body time independent Schrodinger equation. In our calculation, we used the phenomenological Woods-Saxson central potential including spin orbit term in Λ-nucleus interaction and nucleon nucleus interaction. Our calculated single-particle energy levels of Λ-hypernuclei and ordinary nuclei are as shown in table (1). The energy level diagram for Λ-hypernuclei and ordinary nuclei are displayed in figure (1). From Figure (1), it can be seen that the single particle energy level of ${}^{12}_{\Lambda}C$ in s-state is -10.11 MeV. The emulsion experiment is investigated that the single particle energy of ${}^{12}_{\Lambda}C$ in s-state is -10.08 ± 0.18 MeV. Our calculated results by using Woods-Saxson central potential including spin-orbit interaction is in good agreement with the experimental data. The nucleon single particle energy level of ${}^{12}_{6}C$ is -27.5 MeV for s-state and-12.6 MeV and -7.69 MeV for $p_{3/2}$ and $p_{1/2}$ state respectively. There are three bound states for ${}^{12}_{6}C$ and only one bound state for ${}^{12}_{\Lambda}C$. Therefore, the potential depth of Λ-nucleus interaction is shallower than that of nucleon-nucleus interaction.

Table 1: Single-particle energy levels of ${}^{12}_{\Lambda}C$, ${}^{12}_{6}C$

Single particle energy state	${}^{12}_{\Lambda}C$	${}^{12}_{6}C$
$1s_{1/2}$	-10.11 MeV	-27.5 MeV
$1p_{3/2}$		-12.6 MeV
$1p_{1/2}$		-7.69 MeV

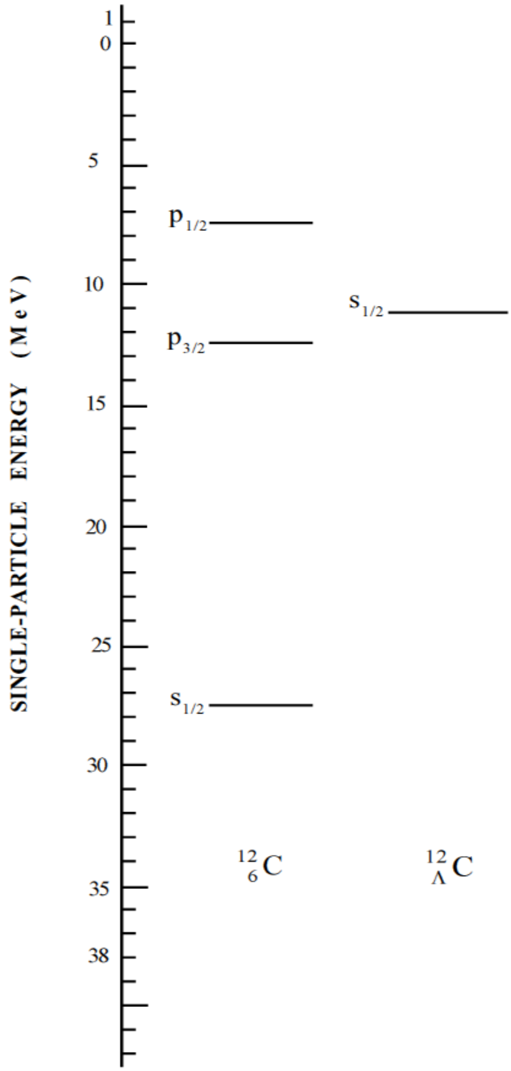

Figure 1: Single-particle energy level of ${}^{12}_{\Lambda}C$, ${}^{12}_{6}C$

Conclusion

We have investigated that the single particle energy levels of $^{12}_{6}C$ and $^{12}_{6}C$ by solving two-body time independent schrodinger equation. Gaussian basis wave function is used in our consideration system. The phenomenological Woods-Saxson potential including spin orbit term is used in Λ nucleus interaction and nucleon nucleus interaction. From our calculated results, it is concluded that the potential strength of Λ-nucleon interaction isabout one third that of nucleon-nucleon interaction.

Acknowledgements

I would like to thank Dr Aung Naing Soe, Pro-Rector, Mandalay University of Distance Education for his encouragement. I am grateful to the full support of Professor Dr Kay thi Nwe, Head of Department of Physics, Mandalay University of Distance Education. I am deeply indebted to Professor Dr Khin Swe Myint, Rector (Rtd), Emeritus Professor, Department of Physics, University of Mandalay for all her enthusiastic discussion, collaboration and encouragement.

References

Danysz, M. Pniewski, J., and Phil, J., Mag. **44** (1953) 348.

Hasegawa, T., et al., Phys. Rev. Lett.**74** (1995) 224.

Henlay, E.M., (1991) “Subatomic Physics”.

Millener, D. J., et al., Phys. Rev. **C31**(1985)499.

Motoba, T.et al.,Nucl. Phys.**A 534** (1991) 597.

Timura, H., et al., Nucl. Phys.**A 639** (1998) 83c.

Vries, H., Jager, C.W and Vries, C. (1987), “Atomic Data and Nuclear Data Tables”, **36**.

Williams, W.S.C., (1991), “Nuclear and Particle Physics”.