

## COUPLED-CHANNELS ANALYSIS FOR HEAVY-ION FUSION REACTIONS OF $^{16}\text{O} + {}^A\text{X}$ SYSTEMS

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### Abstract

In this work, we have studied the fusion cross sections and fusion barrier distributions of heavy-ion collision at energy near and below the Coulomb barrier. The coupled-channels method has been applied to analyze the intrinsic characters of colliding systems. Firstly, we evaluate fusion cross sections for systems having different collective environments. Next, we calculate the fusion cross section and fusion barrier distribution for fixed projectile and different target (i.e.,  $^{16}\text{O} + {}^A\text{X}$  systems where the target has different intrinsic characters such as rotation and vibration). To this end, we compare the experimental fusion cross sections with those from calculations with and without coupling using the same potential parameters of the selected systems. The calculated results indicate the sensitivity of fusion cross sections on types of static and dynamic deformations. The results show that the coupled-channels calculations reproduce the fusion cross sections very well but barrier distribution deviate from the experimental results in the high energy region.

**Keywords:** nuclear structure, rotation, vibration

### Introduction

This study focuses on the fusion cross section and fusion barrier distribution of heavy-ion collisions at energies around the Coulomb barrier are strongly influenced by couplings of the relative motion of the colliding nuclei to several nuclear intrinsic motions. In order to take into account those couplings, the coupled-channels approach has been a standard tool. In this approach, one often uses experimental information on nuclear intrinsic degrees of freedom, such as rotation and vibration. We will apply this approach particularly to the  $^{16}\text{O} + {}^A\text{X}$  systems where the target nuclei have different intrinsic characters, and compare with the experimental results. The structure of colliding nuclei will be investigated from the effects of nuclear intrinsic motion on barrier distribution.

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## Coupled-Channels Formalism for Heavy-Ion Fusion Reactions

In order to take into account excitations of the colliding nuclei during the fusion, we assume the following Hamiltonian:

$$H(\mathbf{r}, \xi) = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r) + H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi), \quad (1)$$

where  $r$  is the coordinate for the relative motion between the projectile and target nuclei, and  $\mu$  is the reduced mass.  $H_0(\xi)$  is the intrinsic Hamiltonian,  $\xi$  representing the internal degrees of freedom.  $V(r)$  is the potential for the relative motion and  $V_{\text{coup}}(\mathbf{r}, \xi)$  is coupling Hamiltonians between the relative motion and intrinsic degrees of freedom, respectively. The coupled-channels equations are obtained by expanding the total wave function in terms of the eigen functions of  $H_0(\xi)$ . This leads to

$$\left[ -\frac{\hbar^2 d^2}{2\mu dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V(r) - E + \epsilon_n \right] u_n^J(r) + \sum_{n'} V_{nn'}(r) u_{n'}(r) = 0, \quad (2)$$

where,  $\epsilon_n$  is the excitation energy for the  $n$ -th channel. In deriving these equations, we have employed the iso-centrifugal approximation and replaced the angular momentum for the relative motion by the total angular momentum,  $J$ . This approximation has been found to be valid for heavy-ion systems, and reduces considerably the dimensions of the coupled-channels problem. We impose the following boundary conditions in solving the coupled-channels equations:

$$u_n^J(r) \rightarrow H_J^{(-)}(k_n r) \delta_{n,0} - \sqrt{\frac{k_0}{k_n}} S_n^J H_J^{(+)}(k_n),$$

(3) Here,  $k_n = \sqrt{2\mu(E - \epsilon_n)/\hbar^2}$  is the wave number for the  $n$ -th channel,

where  $n = 0$  represents the entrance channel.  $S_n^J$  is the nuclear S-matrix, and  $H_J^{(-)}$  and  $H_J^{(+)}$  are the incoming and the outgoing Coulomb wave functions, respectively. Using the S-matrix, the fusion cross sections are calculated as

$$\sigma_F(E) = \frac{\pi}{k_0^2} \sum_J (2J+1) P_J(E), \quad (4)$$

where,  $P_j(E)$  is the penetrability and is related to the nuclear S-matrix by

$$P_j(E) = 1 - \left| \sum_n S_n^J \right|^2.$$

### **Representation of Barrier Distribution**

Extensive experimental as well as theoretical studies have revealed that the inadequacy of the potential model which can be attributed to the effects of the couplings of the relative motion between the colliding nuclei to several nuclear intrinsic motions. Among possible intrinsic excitations of a nucleus, the most relevant nuclear intrinsic motions to heavy-ion fusion reactions are low-lying vibrational excitations with several multi-polarities, or rotational motions of deformed nuclei.

The large enhancement of the fusion cross section, and the strong isotope dependence, are caused by the coupling of the relative motion between the projectile and target to their intrinsic degrees of freedom. The effects of channel coupling can be interpreted in terms of the distribution of fusion barriers and the underlying structure of the barrier distribution can be detected by taking the first derivative of penetrability. In order to point out the several important features of the fusion barrier distribution related with the channel coupling effects, it is important to use the barrier distribution representation. That is, the effects of channel coupling can be interpreted in terms of the distribution of fusion barriers and the underlying structure of the barrier distribution can be detected by taking the first derivative of penetrability is given by

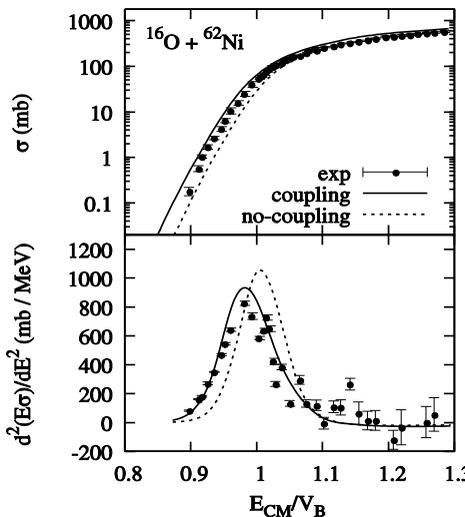
$$D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}. \tag{5}$$

From this expression, it is clear that the first derivative of the product of fusion cross section  $\sigma_f$  and the centre of mass energy  $E$  with respect to the energy,  $d(E\sigma)/dE$ , is proportional to the penetrability of the s-wave scattering. The fusion barrier distributions are obtained with the point difference formula with the energy step of  $\Delta E = 2\text{MeV}$  in order to be consistent with the experimental barrier distribution.

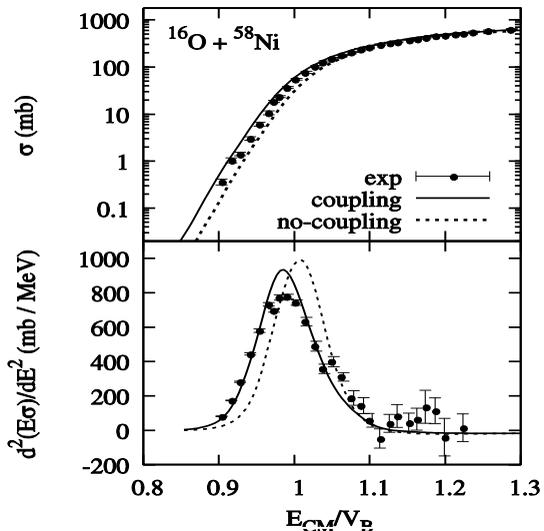
## Results and discussion

This section describes the fusion cross sections and barrier distributions for  $^{16}\text{O} + {}^A\text{X}$  systems (i.e., the same projectile and different targets) such as  $^{16}\text{O} + {}^{58}\text{Ni}$ ,  $^{62}\text{Ni}$ ,  $^{92}\text{Zr}$  and  $^{166}\text{Er}$  reactions are presented. The calculations were performed using the code CCFULL and the relevant parameters used in the calculation are listed in Table (1). The nuclear potential is assumed to have a Woods-Saxon shape and the depth parameter of the potential was chosen to reproduce the experimental fusion barrier heights.

The low lying excitation levels of target nuclei suggest that three of the target nuclei,  $^{58}\text{Ni}$ ,  $^{62}\text{Ni}$  and  $^{92}\text{Zr}$  have vibrational nature and  $^{166}\text{Er}$  have rotational nature. Firstly, the fusion of  $^{16}\text{O}$  and the two Ni isotopes:  $^{16}\text{O} + {}^{58}\text{Ni}$ ,  $^{62}\text{Ni}$  systems are considered and the calculated results are shown in Fig.(1) and (2), respectively. The single quadrupole and octupole phonon coupling to the ( $2^+$ ) and ( $3^-$ ) states taken into account for both  $^{16}\text{O} + {}^{56}\text{Ni}$  and  $^{62}\text{Ni}$  systems. The energies and deformation values of the low lying states are given in Table (2).



**Fig.1.** The results of coupled channels calculations and the corresponding barrier distribution for  $^{16}\text{O} + {}^{62}\text{Ni}$  reaction. The experimental data are taken from [7].

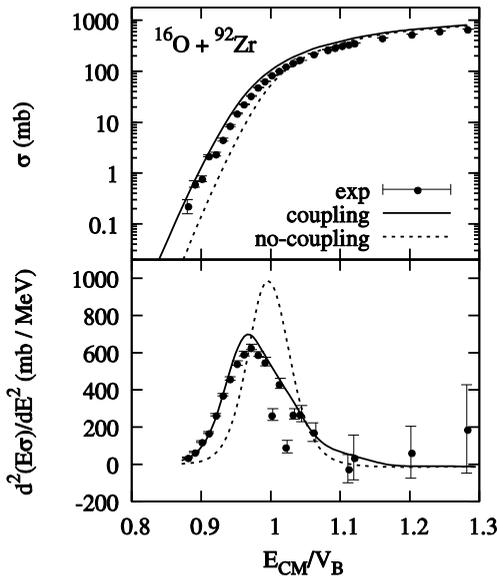


**Fig.2.** The results of coupled channels calculations and the corresponding barrier distribution for  $^{16}\text{O} + {}^{58}\text{Ni}$  reaction. The experimental data are taken from [7].

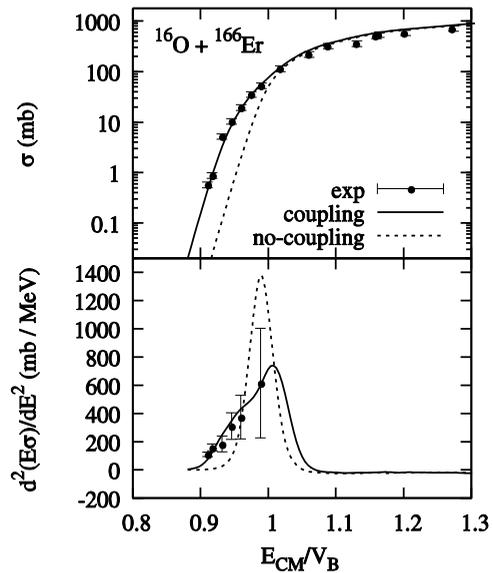
The calculated fusion cross section and fusion barrier distribution are shown in upper and lower panels. The solid lines and dotted lines are calculated results with and without coupling. One can see that the calculated results of the fusion cross sections slightly overestimate the experimental data at low energy region. The extracted fusion barrier distributions well reproduce experimental data although it does not give height of the barrier. The features of the barrier distribution are the same in both results, but the peak positions are slightly shifted to low energy due to the coupling effects.

Next, the reaction of same projectile and heavier target such as  $^{16}\text{O} + ^{92}\text{Zr}$  system will be studied. For fusion with  $^{16}\text{O}$  and vibrational target,  $^{92}\text{Zr}$ , we considered the coupling of one quadrupole phonon to ( $2^+$ ) state and three octupole phonons to ( $3^-$ ) states of the target nucleus,  $^{92}\text{Zr}$ . The results of fusion cross section (upper panel) and barrier distribution (lower panel) are shown in Fig.(3). In the figures, the solid lines show the calculated results with coupling and dotted lines are without coupling. We have performed the calculation with different number of phonons for target nucleus as in the previous section and it is found that the energy dependence of the subbarrier fusion cross section is almost insensitive to the addition of multiphonon excitation of the target but the barrier distribution is strongly sensitive on this effect.

Finally, the fusion reaction of  $^{16}\text{O}$  with heavy deformed nucleus,  $^{166}\text{Er}$ , has been studied. In the calculation, the low lying quadrupole excitation ( $2^+$ ) state of  $^{166}\text{Er}$  and inert projectile,  $^{16}\text{O}$ , are taken into account and the results are displayed by solid lines in Fig.(4). Figure captions are same as above interpretation of fusion cross section and barrier distribution for  $^{16}\text{O} + ^{166}\text{Er}$  system. It is found that the calculated results of fusion cross section and fusion barrier distribution gives good agreement with the experimental data. Thus, we can investigate the structure of heavy elements and get the information of the colliding nuclei from barrier distribution.



**Fig.3:** The results of coupled channels calculations and the corresponding barrier distribution for  $^{16}\text{O} + ^{92}\text{Zr}$  reaction. The experimental data are taken from [8].



**Fig.4:** The results of coupled channels calculations and the corresponding barrier distribution for  $^{16}\text{O} + ^{166}\text{Er}$  reaction. The experimental data are taken from [9].

**Table 1:** The potential parameters used in the coupled channels calculations. The barrier parameters obtained are also given. The values of barrier depth ( $V_B$ ), barrier position ( $R_B$ ) and curvature ( $\hbar\omega$ ) for each system are taken from [7, 8, 9].

Reactions	$V_0$ (MeV)	$r_0$ (fm)	$a$ (fm)	$V_B$ (MeV)	$R_B$ (fm)	$\hbar\omega$ (MeV)
$^{16}\text{O} + ^{58}\text{Ni}$	95.50	1.130	0.610	31.62	9.51	3.92
$^{16}\text{O} + ^{62}\text{Ni}$	98.73	1.135	0.630	30.94	9.74	3.82
$^{16}\text{O} + ^{92}\text{Zr}$	135.00	1.200	0.460	42.44	10.37	5.00
$^{16}\text{O} + ^{166}\text{Er}$	110.50	1.130	0.750	64.70	11.39	4.23

**Table.2: The energies and deformation values of low-lying  $2^+$ ,  $3^-$  states for the different nuclei. The excitation energies and  $\beta(E_2)$  and  $\beta(E_3)$  values are taken from [10]**

<b>Nucleus</b>	<b><math>\lambda</math></b>	<b>E*(MeV)</b>	<b><math>\beta_\lambda</math> (or) <math>\beta_4</math></b>
$^{58}\text{Ni}$	$2^+$	1.454	0.183
	$3^-$	4.474	0.198
$^{62}\text{Ni}$	$2^+$	1.172	0.198
	$3^-$	3.756	0.197
$^{92}\text{Zr}$	$2^+$	0.934	0.101
	$3^-$	2.340	0.280
$^{166}\text{Er}$	$2^+$	0.080	0.238
			0.006( $\beta_4$ )

### **Summary and Conclusion**

In summary, we have used the coupled-channels method to evaluate the effects of coupling for the systems such as  $^{16}\text{O} + ^{58}\text{Ni}$ ,  $^{16}\text{O} + ^{62}\text{Ni}$ ,  $^{16}\text{O} + ^{92}\text{Zr}$  and  $^{16}\text{O} + ^{166}\text{Er}$  reactions. The coupled-channels approach has been applied to investigate heavy-ion fusion reactions at energy near and below the Coulomb barrier. We have discussed the role of collective excitations on barrier distribution using recent experimental data and compared the calculated results of fusion cross section and fusion barrier distribution for  $^{16}\text{O} + ^A\text{X}$  systems with the experimental data. It is found that the fusion cross section and fusion barrier distribution of coupling results well reproduce the experimental data and hence able to investigate the structure of the colliding nuclei. Thus the coupled-channels formalism is very applicable to clearly see the structure of heavy elements in nuclear fusion reactions.

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