SINGLE-PARTICLE ENERGY STATES FOR $^{208}_{\Lambda}$Pb and $^{208}_{82}$Pb

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Abstract

Single-particle energy states in lambda hypernuclei and ordinary nuclei are calculated to study the structures of $^{208}_{82}$Pb and $^{208}_{\Lambda}$Pb nuclei. The phenomenological Woods-Sexon central potential and Woods-Sexon lambda-core nucleus potential including spin orbit interaction are used. Single-particle energy levels are investigated by solving one-body Schrodinger equation with the Gaussian basis treatment.

Keywords: single –particle energy states, Gaussian basis wave function, Woods-Sexon potential, Woods-Sexon lambda-core nucleus potential including spin orbit interaction. *FORTRAN program, Gaussian basis wave function, Woods-Saxon potential*

Introduction

In nuclear physics, the nuclear shell model is a model of the atomic nucleus which uses the Pauli exclusion principle to describe the structure of the nucleus in terms of energy levels. The study of low-lying excited states of closed shell and near-closed shell provide information about specific nuclear orbital nucleus. The purpose of this paper is to determine the energy levels of $^{208}_{82}$Pb and to use the phenomenological Woods-Saxon potential with spin-orbit interaction. It is to concentrate on $^{208}_{82}$Pb nucleus which describes the binding energies with low-lying levels schemes. This determination is the fundamental importance for explaining the structure of nuclei.

The energy levels are found by solving the Schrödinger equation for a single nucleon moving in the average potential generated by all other nucleons. Each level may be occupied by a nucleon, or empty. Some levels accommodate several different quantum states with the same energy, they are said to be degenerate [Greiner, W., A.M. Joachim, 1996].

Some nuclei are bound more tightly than others. This is because two nucleons of the same kind cannot be in the same state. So the lowest energy state of the nucleus is one where nucleons fill all energy levels from the bottom up to some level. The energy levels increase with the orbital angular quantum number l, and s, p, d, f ... symbols are used for l = 0, 1, 2, 3, ... The value of total angular momentum is j and the spin orbit interaction depresses the state with total angular momentum j = l + ½ and rises the one j = l - ½[Frauenfelder, H. and M. H. Ernest, 1972].

Calculation of Single Particle Energy Levels

The time-independent Schrödinger equation for potential well is as follows.

$$\left[\left(-\frac{\hbar^2}{2M}\nabla^2\right)+v(r)\right]\Psi(r) = E\Psi(r) \qquad (1)$$

So, the radial part of Schrödinger equation is as follows.

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$$-\frac{\hbar^2}{2M}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]u(r)+V(r)u(r)=Eu(r) \tag{2}$$

The Gaussian basis wave function is used

$$u(r)=r^{l+1}N\sum_i c_i e^{-(\frac{r}{b_i})^2} \tag{3}$$

These linear equations can be solved by using Gauss elimination method which is the fundamental one for solving linear systems. And so norm matrix elements, kinetic energy matrix element and potential energy matrix elements and root-mean-square distance are analytically obtained by using Gaussian standard integral form. By diagonalzing the Hamiltonian matrix, the energy eigen-value is obtained with the help of FORTRAN PROGRAM.

Normalization Constant, Kinetic Energy, Centrifugal Potential Energy

To solve the physical quantities of $^{208}_{82}Pb$ nuclei, the normalization constant is calculated. The normalized condition is as follows;

$$\int_0^\infty u^*(r)u(r)dr=1 \tag{4}$$

The normalized Gaussian basis wave function is defined as follows;

$$u(r)=A\sum_i c_i e^{-(\frac{r}{b_i})^2} r^{l+1} \tag{5}$$

By using standard Integral, the normalized constant is obtained.

$$A=\frac{1}{\left[\sum_i\sum_j c_i c_j \frac{(2l+1)!!}{2^{l+2}}\frac{1}{(\frac{1}{b_i^2}+\frac{1}{b_j^2})^{l+1}}\left(\frac{\pi}{\frac{1}{b_i^2}+\frac{1}{b_j^2}}\right)^{1/2}\right]^{1/2}} \tag{6}$$

The kinetic energy is calculated as follows;

$$\hat{T}(r)u(r)=\frac{\hbar^2}{2\mu}A\sum_i c_i e^{-(\frac{r}{b_i})^2}\left[l(l+1)r^{l-1}-\frac{2}{b_i^2}(2l+3)r^{l+1}+\frac{4}{b_i^4}r^{l+3}\right] \tag{7}$$

The average kinetic energy is as follows.

$$\left\langle\hat{T}(r)\right\rangle=\frac{\hbar^2}{2\mu}A^2\sum_i\sum_j c_i c_j\left[(\ell^2+\ell)\frac{(2\ell-1)!!\sqrt{\pi}}{2^{\ell+1}}\frac{1}{(\frac{1}{b_i^2}+\frac{1}{b_j^2})^{\ell+0.5}}\right. \tag{8}$$

$$\left.-\frac{2}{b_j^2}(2\ell+3)\frac{(2\ell+1)!!\sqrt{\pi}}{2^{\ell+2}}\frac{1}{(\frac{1}{b_i^2}+\frac{1}{b_j^2})^{\ell+1.5}}+\frac{4}{b_j^4}\frac{(2\ell+3)!!\sqrt{\pi}}{2^{\ell+3}}\frac{1}{(\frac{1}{b_i^2}+\frac{1}{b_j^2})^{\ell+2.5}}\right]$$

The centrifugal potential energy is follows;

$$\hat{F}(r)u(r) = \frac{\hbar^2}{2\mu}\left[\frac{l(l+1)}{r^2}u(r)\right] \tag{9}$$

By using wave function, the centrifugal potential energy is described as follows;

$$\hat{F}(r)u(r) = \frac{\hbar^2}{2\mu}\left[\frac{l(l+1)}{r^2}\, A\sum_i c_i e^{-(\frac{r}{b_i})^2} r^{l+1}\right] \tag{10}$$

The average centrifugal potential energy is calculated.

$$\left\langle \hat{F}(r)\right\rangle = \frac{\hbar^2}{2M}\, A^2 \sum_i \sum_j c_i c_j (\ell^2+\ell)\frac{(2\ell-1)!!\sqrt{\pi}}{2^{\ell+1}}\frac{1}{(\frac{1}{b_i^2}+\frac{1}{b_j^2})^{\ell+0.5}} \tag{11}$$

The Lambda-Core Nucleus Interaction

The interactions between nuclei are commonly described by using phenomenological Woods-Saxon potential that plays a great role in nuclear physics. Woods-Saxon potential is based on the sum of a spin-independent central potential and a spin-orbit potential.

$$V(r) = V_0(r) + V_{ls}(r)\ \vec{l}.\vec{s} \tag{12}$$

The form of the generalized spin-independent central Woods-Saxon potential is as follows.

$$V_0(r) = -\frac{V_0}{1+e^{(r-R)/a}} \tag{13}$$

$R = r_0 A^{1/3}$ is the nuclear radius where r_0 = 1.25 fm and A is the mass number.

Typical values for the parameters are: $V_0 \approx 50\,\text{MeV}$, $a \approx 0.53\,\text{fm}$ [Woods, R.D. and D.S. Saxon, 1954].

For lambda particle, $V_0 \approx 30\,\text{MeV}$, $a \approx 0.6\,\text{fm}$ and r_0 = 1.1 fm are used [M. Wang, et al., 2014].

The potential strength depends upon the number of proton and neutron. It is represent by,

$V_0 = \left[50 - 32\frac{N-Z}{A}\right]$ and the nuclear density, $\rho(r) = \frac{1}{1+e^{r-R/a}}$.

Woods-Saxon potential including spin-orbit interaction is

$$V(r) = V_0(r) + V_{ls}(r) \tag{14}$$

The spin-orbit interaction is described as follows.

$$V_{ls}(r) = V_{so}(\frac{\hbar}{Mc})^2 (l.s)\,\frac{1}{r}\,\frac{d\rho}{dr} \tag{15}$$

The total angular momentum is $\vec{j} = \vec{l} + \vec{s}$ and the spin of a nucleon is $\frac{1}{2}\hbar$.

For $j = l + \frac{1}{2}$, $l.s = \frac{1}{2}l$

$$\therefore\ V(r) = V_0(r) + V_{ls}(r) \tag{16}$$

$$= \frac{-V_0}{1+e^{r-R/a}} + V_{so}(\frac{\hbar}{Mc})^2(\frac{1}{2}l)\left[\frac{1}{r}\frac{e^{r-R/a}}{1+e^{r-R/a}}\frac{1}{a}\right]$$

For $j = l - \frac{1}{2}$, $l.s = -\frac{1}{2}(l+1)$

$$\therefore V(r) = \frac{-V_0}{1+e^{r-R/a}} - V_{so}(\frac{\hbar}{Mc})^2(\frac{1}{2}(l+1))\left[\frac{1}{r}\frac{e^{r-R/a}}{1+e^{r-R/a}}\frac{1}{a}\right] \quad (17)$$

So, the average Woods-Saxon potential with spin orbit interaction, for $j = l + \frac{1}{2}$ state is as follow,

$$\langle V(r)\rangle = \langle V_{W-S}(r)\rangle + \langle V_{\ell s}(r)\rangle$$
$$= A^2\sum_i\sum_j c_i c_j \left[r^{2\ell+2} e^{-(\frac{r^2}{b_i^2+b_j^2})}\left(\frac{-V_0}{1+e^{(r-R)/a}}\right)\right.$$
$$\left. - V_{so}\left(\frac{\hbar^2}{m_\pi c}\right)\frac{1}{a} r^{2\ell+1} e^{-(\frac{r^2}{b_i^2+b_j^2})}\frac{1}{2}\ell\frac{e^{(r-R)/a}}{(1+e^{(r-R)/a})^2}\right] \quad (18)$$

So, the average Woods-Saxon potential with spin orbit interaction, for $j = l - \frac{1}{2}$ state is as follow,

$$\langle V(r)\rangle = A^2\sum_i\sum_j c_i c_j \left[r^{2\ell+2} e^{-(\frac{r^2}{b_i^2+b_j^2})}(-\frac{V_0}{1+e^{(r-R)/a}})\right.$$
$$\left. -V_{so}(\frac{\hbar}{m_\pi c})^2\frac{1}{a} r^{2\ell+1} e^{-(\frac{r^2}{b_i^2+b_j^2})}\frac{1}{2}(-\ell-1)\frac{e^{(r-R)/a}}{(1+e^{(r-R)/a})^2}\right] \quad (19)$$

Root-Mean Square Distance

The root-mean square distance of a particle is as follows.

$$\langle r^2\rangle = \int \psi(r)^* r^2 \psi(r)\, dr \quad (20)$$

By using normalized wave function, the root-mean square distance is described as follows.

$$\langle r^2\rangle = A^2\sum_i c_i\sum_j c_j \frac{(2l+3)(2l+3-2)!!}{2^{l+2}\times 2}\frac{1}{(\frac{1}{b_i^2}+\frac{1}{b_j^2})^{l+1}}\frac{1}{(\frac{1}{b_i^2}+\frac{1}{b_j^2})}(\frac{\pi}{\frac{1}{b_i^2}+\frac{1}{b_j^2}})^{1/2} \quad (21)$$

Results

The calculated results of energy levels of $^{208}_{82}Pb$ and $^{208}_{\Lambda}Pb$ are shown in Table 1 and Table 2 and then the corresponding energy shell levels are shown in Fig. 1 and Fig. 2 respectively. In these figures, the innermost level $1s_{1/2}$ has the highest binding energy. The

binding energy of the single particle $1s_{1/2}$ state is gradually decreased to $1i_{11/2}$ levels away from the $^{208}_{82}\mathrm{Pb}$ nucleus. The values of total energy decrease with the increasing of orbital angular momentum. In spin-orbit interaction, a single particle has a doublet state. The splitting of a single particle has $\mathrm{j}=l+\frac{1}{2}$ and $\mathrm{j}=l-\frac{1}{2}$. It is found that $l+\frac{1}{2}$ state is the lower state and more bounds than the upper level, $l-\frac{1}{2}$ state.

Discussion

By comparing the calculated results of energy levels of $^{208}_{\Lambda}\mathrm{Pb}$ and $^{208}_{82}\mathrm{Pb}$, the energy levels of $\wedge$-hypernucleus is smaller than the ordinary nuclei and the energy of $^{208}_{\Lambda}\mathrm{Pb}$ nuclei is looser than the $^{208}_{82}\mathrm{Pb}$ nuclei. Then, the level splitting between $1p_{1/2}$ and $1p_{3/2}$ for $^{208}_{\Lambda}\mathrm{Pb}$ is 0.66 MeV and that for $^{208}_{82}\mathrm{Pb}$ is 0.28 MeV respectively.

The energy of $^{208}_{\Lambda}\mathrm{Pb}$ nucleus at s-state is calculated and it is -25.29MeV and it is nearly equal to experimental result, -26.3 MeV [Chhanda ,S., and A.Thomas, 2018].

The values of root-mean-square distances decrease with the total binding energies of $^{208}_{\Lambda}\mathrm{Pb}$ and $^{208}_{82}\mathrm{Pb}$ nucleus due to the $\vec{l}$ and $\vec{\mathrm{s}}$ interaction. The innermost portion of these two nuclei has zero potential and the higher binding energy of $l+\frac{1}{2}$ state is the outer side of the lower binding energy of $l-\frac{1}{2}$ state. So, the root-mean-square distance of $l+\frac{1}{2}$ state is greater than that of $l-\frac{1}{2}$ state.

Conclusion

The Woods-Saxon potential is a convenient phenomenological choice for the one body potential. It provides a model for the properties of bound-state and continuum single-particle wave functions.

Table 1 The Physical Quantities of $^{208}_{82}\mathrm{Pb}$

Single-Particle State of $^{208}_{82}\mathrm{Pb}$	**Total Energy (MeV)**	**Root-Mean-Square Distance(fm)**
$1s_{1/2}$	-46.09	3.94
$1p_{3/2}$	-42.25	4.63
$1p_{1/2}$	-41.97	4.56
$1d_{5/2}$	-37.64	5.11
$1d_{3/2}$	-36.93	4.99
$1f_{7/2}$	-32.37	5.48
$1f_{5/2}$	-30.99	5.32
$1g_{9/2}$	-26.50	5.79
$1g_{7/2}$	-24.21	5.58
$1h_{11/2}$	-20.11	6.07
$1h_{9/2}$	-16.67	5.82
$1i_{13/2}$	-13.26	6.32
$1i_{11/2}$	-8.43	6.05

Table 2 The Physical Quantities of $^{208}_{\Lambda}\mathrm{Pb}$

Single-Particle State of $^{208}_{\Lambda}\mathrm{Pb}$	**Total Energy (MeV)**	**Root-Mean-Square Distance(fm)**
$1s_{1/2}$	-25.29	3.60
$1p_{3/2}$	-20.97	4.39
$1p_{1/2}$	-20.32	4.20
$1d_{5/2}$	-15.99	4.94
$1d_{3/2}$	-14.43	4.76
$1f_{7/2}$	-10.54.	5.40
$1f_{5/2}$	-7.68	5.19
$1g_{9/2}$	-4.64	5.85
$1g_{7/2}$	-0.35	5.79

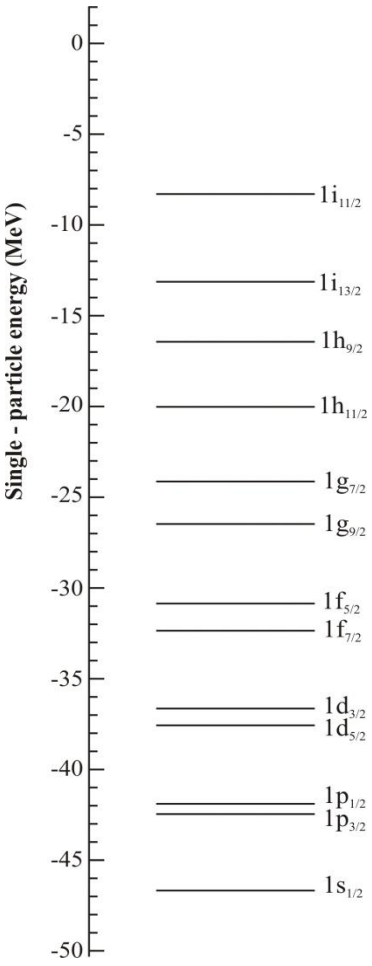

Figure 1 Single-particle energy levels of $^{208}_{82}$Pb

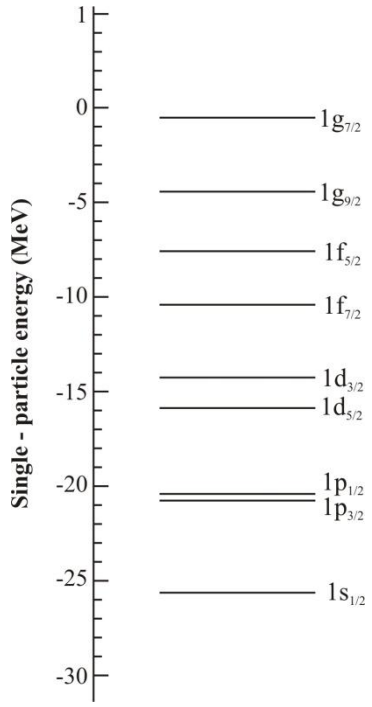

Figure 2 Single-particle energy levels of $^{208}_{\Lambda}$Pb

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