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THEORETICAL INVESTIGATION ON Λ (1405) RESONANCE WITHIN $\bar{K}N$ FRAME WORK OF $\bar{K}N-\Sigma^+\pi^-$ COUPLED CHANNEL

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Abstract

We calculated various range parameters and different strength parameters of Λ(1405) with resonance state for $\Sigma^+\pi^- \to \Sigma^+\pi^-$. Firstly we solved Schrodinger equation by using separable potential for $\Sigma^+\pi^- \to \Sigma^+\pi^-$ channel to obtain various differential cross sections with energies for various strength parameters and different range parameters. It is reduced the resonance state of $\Sigma^+\pi^-$. Secondly, we investigated the various strength and range parameters of seperable potential for Λ(1405) resonance within $\bar{K}N$ frame work of $\bar{K}N \to \Sigma^+\pi^-$ coupled channel. It is observed that the parameter sets of $\bar{K}N$ interaction can be constructed for $\Sigma^+\pi^-$ resonance and K^-p bound state. Therefore, we constructed the new model **A** and **B** for $\bar{K}N$ interaction.

Keyword: Resonance state, coupled channel and $\bar{K}N$ interaction.

Introduction

1.1 Reviews of Theoretical Investigation and Experimental Observation

The resonant state of Λ(1405) with J = 1/2, I = 0, S = -1, called Λ (1405), is located below the $\bar{K}N$ threshold, and decays to $\Sigma\pi$. The chiral dynamics theories suggested two poles in the coupled $\bar{K}N-\Sigma\pi$ scheme and they determined Λ(1405) with level width 120 MeV , to which counter arguments were given. More recently, J. Esmaili et al. analyzed old bubble-chamber data of stopped- K^- on ^{4}He with a resonant capture process, and found the best-fit value to be $M = 1405.5^{+1.4}_{-1.0}$ MeV/c^2. Hassanvand et al. analyzed recent data of HADES on, $p+p \to p+K^+ + \Lambda(1405)$, and subsequently deduced $M = 1405^{+11}_{-9}\,\text{MeV}/c^2$ and $\Gamma = 62 \pm 10\,\text{MeV}$. Now, the new PDG values have been revised to be $M = 1405.1^{+1.3}_{-1.0}$MeV/c^2 and $\Gamma = 50.5 \pm 2.0$MeV, upon adopting the consequences of these analyses.

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Therefore, we investigated a model $\bar{K}N$ quasi-bound state by changing the strength and range of the $\bar{K}N$ interaction. The purpose of this work is to perform theoretical investigation on a Λ(1405) within $\bar{K}N$ frame work of $\bar{K}N$ - $\Sigma^+\pi^-$ coupled channel.

Resonance State in Single Channel

2.1 Calculation of Resonance State for Single Channel

The Λ(1405) resonance is an I=0 quasi-bound state of $\bar{K}N$, which is embedded in continuum of $\sum\pi$ as a kind of Feshbach resonances. A model for low-energy meson-baryon interaction in the strange sector is presented. The interaction is described in terms of separable potentials with multiple partial waves considered.

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(\mathbf{r}) + v(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad (1)$$

Separable potential is $v(\mathbf{r},\mathbf{r}') = g(r)Y_{00}\frac{\bar{v}_0}{b^3}g(r')Y_{00}$ (2)

The time independent Schrodinger equation can be written as

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(\mathbf{r}) + \frac{\bar{v}_0}{b^3}g(r)Y_{00}\int g(r')Y_{00}\psi(\mathbf{r}')d\mathbf{r}' = \frac{\hbar^2}{2\mu}k^2\psi(\mathbf{r}) \quad (3)$$

$$-\frac{d^2}{dr^2}u(r) + G_0 r\, g(r)\int_0^{\alpha} g(r')u(r')r'dr' = k^2u(r) \quad (4)$$

$$\text{Where, } G_0 \equiv \frac{2\mu}{\hbar^2}\bar{v}_0\frac{1}{b^3} \quad (5)$$

$$\text{Where, } G \equiv G_0\int_0^{\alpha} g(r')u(r')r'dr' \quad (6)$$

We employ Yukawa type form factor as $g(r) = \frac{b}{r}e^{-\frac{r}{b}}$ (7)

The inhomogeneous equation is $-\frac{d^2}{dr^2}u_0(r) + G.r\, g(r) = k^2u_0(r)$ (8)

The solution of inhomogeneous equation is $u_0(r) = Ae^{-\frac{r}{b}}$

The solution of homogeneous equation is $u_1(r) = a\sin(kr + \delta)$

From equation (8), we get the equation as

$$\frac{G}{G_0} = -\frac{1}{2}b^2 a\sin\delta + a\frac{kb}{1+k^2b^2}b^2\cos\delta + a\frac{1}{1+k^2b^2}b^2\sin\delta \qquad (9)$$

By solving equation (10)

$$\sigma_{tot} = \frac{4\pi}{k^2}\frac{k^2b^2}{k^2b^2 + \frac{1}{4}\left\{1 - k^2b^2 + \frac{1}{S}(1+k^2b^2)^2\right\}^2} \qquad (10)$$

Where, $S = \frac{1}{2}G_0 b^5 = \frac{1}{2}\frac{2\mu}{\hbar^2}\overline{v}_0 b^2$, S = strength parameter $B_0 = \frac{\hbar^2}{2\mu}\frac{1}{b^2}$,

B_0 = dynamic pole parameter, b = potential range parameter.

We solved numerically equation (10) by using FORTRAN CODE to obtain resonance state. The results are shown in next section.

2.2 Bound and Resonance Pole

The single channel Schoredinger eqation is

$$-\frac{d^2}{dr^2}u_0(r) + G.rg(r) = k^2u_0(r), \qquad (11)$$

The solution of equation is as equation (11).

$$A = \frac{Gb^3}{1+k^2b^2} = \frac{Gb^3}{(1+ikb)(1-ikb)} \qquad (12)$$

$$E_B = -B_0\left\{\sqrt{|S|} \pm 1\right\} \qquad (13)$$

We can find bound state from equation (13).

For resonance state

$$E_{Res} = E_R - i\frac{\Gamma}{2} \qquad (14)$$

$$E_R = B_0(S-1) \tag{15}$$

$$\frac{\Gamma}{2} = \pm 2B_0\sqrt{S} \tag{16}$$

We calculated resonance states and level widths of $\Sigma^+\pi^-$ for various dynamical poles and strength parameters by using equation (15) and (16).

Resonance and Bound State in Coupled Channel

We consider two channels of $\overline{K}N$ (K^-p) and $\pi\Sigma$ ($\pi^-\Sigma^+$) for simplicity. We employ a set of separable potentials with a Yukawa-type form factor, $\overline{K}N$ channel, 1, or the $\pi\Sigma$ channel, 2, μ_I and μ_{II} are the reduced mass of the channel 1and 2.

$$\begin{aligned} -\frac{d^2}{dr^2}u_I(r) + (G_I + G_{III})r\,g(r) &= k^2 u_I(r), \\ -\frac{d^2}{dr^2}u_{II}(r) + (G_{II} + G_{III})r\,g(r) &= (k^2-\Delta^2)u_{II}(r) \end{aligned} \tag{17}$$

Where, $g(r) = \frac{b}{r}e^{-\frac{r}{b}}$, $\Delta^2 = \frac{2\mu_{II}}{\hbar^2}(M_p + m_k - M_{\Sigma+} - m_{\pi-}) = 1.711\text{fm}^{-2}$

$$k' = \sqrt{\frac{\mu_{II}}{\mu_I}}k = 1.609k$$

The coupled-channel equation for the radial wave functions, $u_1(r)$ and $u_2(r)$, of the present interaction model is written as follows:

$$\begin{cases} u_I(r) = \dfrac{(G_I + G_{II})b^3}{1+k^2b^2}\left(e^{-\frac{r}{b}} - e^{ikr}\right) \\ u_{II}(r) = \dfrac{(G_{II} + G_{III})b^3}{1-(k^2-\Delta^2)b^2}\left(e^{-\frac{r}{b}} - e^{-\sqrt{\Delta^2-K^2}r}\right) \end{cases} \tag{18}$$

$$\begin{cases} G_{I} = -S_{I}\dfrac{G_{I}+G_{II}}{1+k^2b^2}\dfrac{1+ikb}{1-ikb} \\ G_{III} = -S_{III}\dfrac{G_{II}+G_{III}}{1-(\Delta^2-k^2)b^2}\dfrac{1-\sqrt{\Delta^2-k^2}b}{1+\sqrt{\Delta^2-k^2}b} \\ G_{II} = -S_{II}\dfrac{G_{II}+G_{III}}{1-(\Delta^2-k^2)b^2}\dfrac{1-\sqrt{\Delta^2-k^2}b}{1+\sqrt{\Delta^2-k^2}b} \\ G_{III} = -S_{III}\dfrac{G_{I}+G_{III}}{1+k^2b^2}\dfrac{1+ikb}{1-ikb} \end{cases} \tag{19}$$

$$\begin{cases} S_{I} = \dfrac{1}{2}\dfrac{2\mu_I}{\hbar^2}\overline{V}_0^{I}b^2 \\ S_{III} = \dfrac{1}{2}\dfrac{2\mu_I}{\hbar^2}\overline{V}_0^{III}b^2 \\ S_{II} = \dfrac{1}{2}\dfrac{2\mu_{II}}{\hbar^2}\overline{V}_0^{II}b^2 \\ S_{III} = \dfrac{1}{2}\dfrac{2\mu_{II}}{\hbar^2}\overline{V}_0^{III}b^2 \end{cases} \tag{20}$$

$$\left\{1+\frac{S_I}{(1-ikb)^2}\right\}\left\{1+\frac{S_{II}}{(1+\sqrt{\Delta^2-k^2}b)^2}\right\} = \frac{S_{III}\,S_{III}}{(1+\sqrt{\Delta^2k^2}b)^2(1-ikb)^2}$$

$$\left\{(1-ikb)^2+S_I\right\}\left\{(1+\sqrt{\Delta^2-k^2}b)^2+S_{II}\right\} = S_{III}S_{III} \tag{21}$$

For uncoupled case,

$$\left\{(1-ikb)^2+S_I\right\}\left\{(1+\sqrt{\Delta^2-k^2}b)^2+S_{II}\right\} = 0 \tag{22}$$

Resonance state for $S_I > 0$

$$E_R = \frac{\hbar^2}{2\mu_I}k^2 = B_0^{(I)}(S_I-1) - i\,B_0^{(1)}2\sqrt{S_I} \tag{23}$$

Bound state for $S_{II} < 0$

$$E_B = \frac{\hbar^2}{2\mu_{II}}k^2 = \frac{\hbar^2}{2\mu_{II}}\Delta^2 - B_0^{(II)}(\sqrt{|S_{II}|}-1)^2 \tag{24}$$

3.1 Numerical model A

We assumed that the range parameter of Yukawa type separable potential is to be $b=\frac{\hbar C}{M_B C^2}=0.25\text{fm}$, $M_B=789\text{Me}\overline{\text{V}}/C^2\sim\rho\,\text{meson mass}$ (25)

$$\begin{cases} B_0^{(I)}=\frac{\hbar^2}{2\mu_I}\frac{1}{b^2}=2494\text{Me}\overline{\text{V}} \\ B_0^{(I)}=\frac{\hbar^2}{2\mu_{II}}\frac{1}{b^2}=963.2\text{Me}\overline{\text{V}} \end{cases} \tag{26}$$

Bound state in channel II

$$BE_{II}=B_0^{(II)}(\sqrt{|S_{II}|}-1)^2=27\,\text{Me}\overline{\text{V}}\sim\Delta(1405) \tag{27}$$

S_{II} = -1.363

$$E_B=\frac{\hbar^2}{2\mu_{II}}k^2=(103-27)-120\text{Me}\overline{\text{V}} \tag{28}$$

$$k=0.7042-i\,0.09111\text{fm}^{-1} \tag{29}$$

$$S_I S_{II}+1.383S_I+0.9239\,S_{II}+1.325=C^2 \tag{30}$$

If S_I=0, S_{II} = -1.013, C^2=0.3891

Resonance state in channel I.

$$f(z)=\left\{(1-ibz)^2+S_I\right\}\left\{1+b\sqrt{\Delta^2-a^2z^2})^2+S_{II}(S_I)\right\}-C^2(Sz) \tag{31}$$

$$f'(z)=-2ib(1-ibz)\left\{1+b\sqrt{\Delta^2-a^2z^2})^2+S_{II}\right\}$$

$$-2\left\{(1-ibz)^2+SI\right\}(1+b\sqrt{\Delta^2-a^2z^2})^2\frac{a^2bz}{\sqrt{\Delta^2-a^2z^2}} \tag{32}$$

$$Z^{(n+1)}=Z^{(n)}-f(z^{(n)})/f'(Z^{(n)}) \tag{33}$$

We solved eq: (58) numerically by using FORTRAN CODE to obtain resonance energy of $\Sigma\pi$.

3.2 Numerical model B

$$b = \frac{\hbar C}{M_B C^2} = 3.5\text{fm}, \quad M_B = 56.4\text{MeV}/C^2 \tag{34}$$

$$\begin{cases} B_0^{(I)} = \dfrac{\hbar^2}{2\mu_I}\dfrac{1}{b^2} = 12.7\text{MeV} \\ B_0^{(II)} = \dfrac{\hbar^2}{2\mu_{II}}\dfrac{1}{b^2} = 4.91\text{MeV} \end{cases} \tag{35}$$

Bound state in Channel II uncoupled

$$BE_{II} = B_0^{(II)}\left(\sqrt{|S_{II}|} - 1\right)^2 = 27\text{MeV}$$

$$S_{II} = -11.2 \tag{36}$$

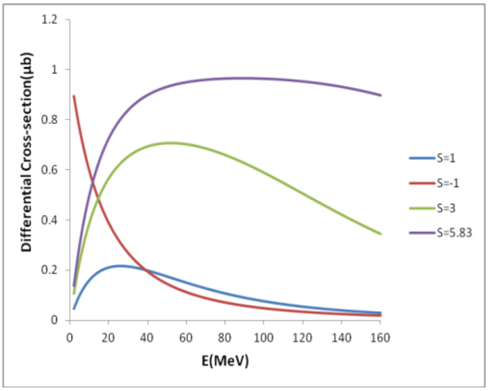
In this model, we solved eq:(58) numerically by using FORTRAN CODE to obtain potential strength and range parameters.

Results and Discussion

4.1 Resonance state for $\Sigma^+\pi^- \rightarrow \Sigma^+\pi^-$

We calculated resonance state for $\Sigma^+\pi^- \rightarrow \Sigma^+\pi^-$ by solving Schrodinger equation. The differential cross sections with different $\Sigma^+\pi^-$ energies are obtained by changing strength parameters for fixed range parameters.

Firstly we calculated various differential cross-sections for various strength parameters at range parameter 2 fm, 3.5 fm and 5.0 fm. The results are shown in Figure (4.1), (4.2) and (4.3). It is observed that pole positions are varied with strength parameters. The various resonance states $\Sigma^+\pi^-$ are obtained from various strength parameters with dynamical-pole parameter 39.0MeV, 12.7 MeVand 6.2 MeV. The results are shown in table (4.1).



Figure(4.1): Various differential cross sections and energies of $\Sigma^{+}\pi^{-}$ for different strength parameters with fixed range parameter b=2 fm

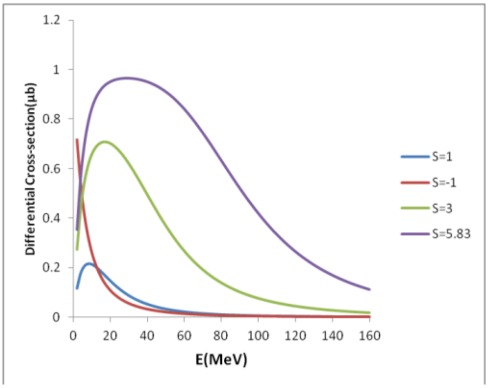


Figure (4.2): Various differential cross sections and energies of $\Sigma^{+}\pi^{-}$ for different strength parameters with fixed range parameter b=3.5 fm

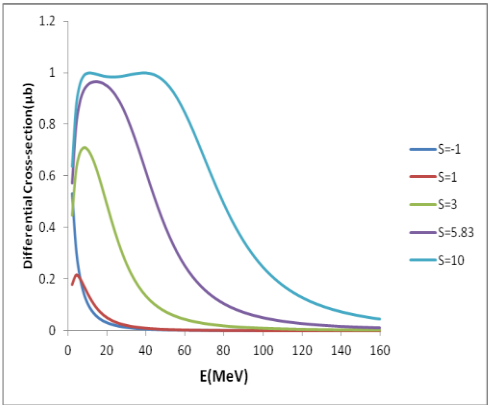


Figure (4.3): Various differential cross sections and energies of $\Sigma^{+}\pi^{-}$ for different strength parameters with fixed range parameter b=5 fm

Table (4.1): Resonance energies and level widths of $\Sigma^{+}\pi^{-}$ for various strength parameters and fixed range parameters for single channel

Range Parameter b(fm)	Strength Parameter (S)	Resonance Energy E_R (MeV)	Level Width Γ (MeV)
2.0	-1	-77.95	-
	1.0	0	155.9
	3.0	77.95	270.03
	5.83	188.25	376.43
3.5	-1	-25.45	-
	1.0	0	50.91
	3.0	25.45	88.17
	5.83	61.47	122.91
5.0	-1	-12.47	-
	1.0	0	24.94
	3.0	12.47	43.20
	5.83	30.12	60.23
	10.0	56.12	78.88

4.2 Resonance state and Bound State from coupled channel

We consider two channels of $\bar{K}N$ ($K^{-}p$) and $\pi\Sigma$ ($\pi^{-}\Sigma^{+}$) for simplicity. We employ a set of separable potentials with a Yukawa-type form factor. The parameter sets of separable potential for bound state of $K^{-}p$ and resonance state of $\pi\Sigma$ are obtained by solving coupled channel. We constructed the numerical model A and model B for parameters of separable potential. The results are shown in the following table.

Table (4.2) Resonance energies and level widths of $\Sigma^{+}\pi^{-}$ for various strength and fixed range parameters for coupled channel

Our Model	Range Parameter b(fm)	Bound state in channel 2		Resonance state in coupled channel			Level Width Γ (MeV)
		Strength Parameters	Bound state energy of pK^- (MeV)	Strength Parameters (coupled case)		Resonance energy of $\Sigma\pi$ (MeV)	
				S_I	S_{II}		
A	0.25	-1.013	27.0	0.925	-0.642	76.0 58.5	40.0 8450
B	3.5	-11.2	27.0	5.8	-11.14	76.0 60.3	40.0 77.0

Conclusion

We investigated various differential cross sections with different $\Sigma^{+}\pi^{-}$resonance energies at various strength and fixed range parameters for single channel. The resonance energy for range parameter 3.5 fm is 61.47 MeV and level width is 122.91 MeV . It is agreement with D. Jido et.al result [1]. The strength and range parameter sets of $\overline{K}N$ interaction for bound state of pK^- and resonance state of $\Sigma^{+}\pi^{-}$are obtained on Λ (1405) resonance and bound state from $K^{-}p \rightarrow \Sigma^{+}\pi^{-}$ coupled channel calculation. We can construct our new model A and B for $\overline{K}N$ interaction. It is observed that separable potential is Yukawa type which can be solved analytically.

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