NEUTRON SINGLE PARTICLE ENERGY STATES IN ^{118}Sn BY APPLYING WOODS-SAXON POTENTIAL WITH SPIN ORBIT INTERACTION

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Abstract

The purpose of this research work is to analyze the neutron single particle energy levels of ^{118}Sn in non-relativistic shell model. The single particle energy levels and wave function of ^{118}Sn nucleus are obtained by solving one body Schroedinger equation numerically with Numerov Method. Numerov method is one of the methods which can give the energy and wave function simultaneously. In our calculation, the Woods-Saxon potential with spin orbit term is used for interaction between core and single nucleon. . In our calculation of ^{118}Sn, the calculated shell structure of neutron single particle are $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$, $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$.

Keywords: single particle energy, spin-orbit interaction, angular momentum

Introduction

The determination of the single particle energies of the atomic nuclei is an important problem in nuclear physics. An analytic expression for single-particle energy levels in the nucleus is based on a phenomenological behavior of main feature of shell model. The nuclear shell model is founded on the principle that neutrons and protons can move as independent in orbitals with discrete quantum numbers. In a physics of nuclear structure, a single particle state is assumed as an excitation state that one proton or one neutron jumped to a higher orbit. The term single particle state is usually used in the analysis of non-interacting or weakly interacting particles.

In our research work, the neutron single particle energy levels and wave function of the ^{118}Sn (Tin) nucleus are obtained by solving one body Schrödinger equation numerically with Numerov method. Tin has the largest number of stable isotopes. We have investigated the single particle levels of ^{118}Sn by using Woods-Saxon potential with spin orbit term for interaction between core and single nucleon.

Theoretical framework

In order to investigate the nucleon single-particle energy levels in^{118}Sn, we firstly solved one-body Schroedinger equation by applying Numerov method.

The Schroedinger radial equation is

$$\frac{d^2u(r)}{dr^2}+\frac{2m}{\hbar^2}\left[(E-V(r)-\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u(r)=0 \qquad (1)$$

We can integrate equation (1) by means of the Numerov method.

In Numerov method, the forward and backward recursive relations are examined by using Taylor series expansion method.

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Firstly it is split that the 'r' range into N points according to "$r_n = r_{n-1} + h$"

By using Taylor series expansion,

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2 \frac{f''(a)}{2!} + ---- + \frac{(x-a)^{n-1} f^{n-1}(a)}{(n-1)!} \tag{2}$$

(i) forward recursive relation

$$[1+\frac{h^2}{12}k(r_{n-1})]u(r_{n-1}) = 2[1-\frac{5h^2}{12}k(r_n)]u(r_n) - [1+\frac{h^2}{12}k(r_{n+1})]u(r_{n+1}) \tag{3}$$

(i) backward recursive relation

$$u(r_{n-1}) = \frac{2[1-\frac{5h^2}{12}k(r_n)]u(r_n) - [1+\frac{h^2}{12}k(r_{n+1})]u(r_{n+1})}{[1+\frac{h^2}{12}k(r_{n-1})]} \tag{4}$$

According to the properties of wave function, u_{out} and u_{in} must be homogeneous at a point r_c called matching point. According to continuity equation

$$(u_{out})_{r_c} = (u_{in})_{r_c} \tag{5}$$

$$(u'_{out})_{r_c} = (u'_{in})_{r_c} \tag{6}$$

We can define a function Match (E) at r_c whose zeros correspond to the energy eigen values as

$$\text{Match }(E) = \left[\frac{u'_{out}}{u_{out}}\right]_{r_c} - \left[\frac{u'_{in}}{u_{in}}\right]_{r_c} \tag{7}$$

$$E_n = E_{n-1} + \Delta E \tag{8}$$

For each E_n, we have to calculate their Eigen function u_{out} and u_{in} at r_c point. Then the match point, r_c, have been defined and this matching point is within the N points which covered the interaction range. The matching point was selected at N/3. The trial energy range splitting have been set into N points according to $E_n = E_{start} + E$. For each E_n, the Eigen function u_{out} (r) and u_{in} (r) have been calculated at the r_c point. We build a "Match (E)" function based on continuity condition. Finally, the correct energy Eigen value (E) and Eigen function have been searched simultaneously.

In order to calculate nucleon single particle energy levels in $_{50}Sn^{118}$ for different angular momentum states, phenomenological Woods-Saxon potential is used. It is based on the sum of spin independent-central potential and a spin orbit potential.

Woods-Saxon potential with spin-orbit coupling term is express as follows.

$$V_{so}(r) = V_{so}\frac{1}{r}\frac{d}{dr}f_0(r)(\vec{l}.\vec{s}) = -V_{so}\frac{1}{r}\frac{1}{a_0}\frac{\exp[(r-R_0)/a_0]}{\left[1+\exp\left[\frac{(r-R_0)}{a_0}\right]\right]^2}(\vec{l}.\vec{s}) \tag{9}$$

where, V_{so} = the strength of Woods-Saxon spin-orbit coupling potential term

Results and discussion

We have investigated the neutron single-particle energy levels of ^{118}Sn by solving one-body Schroedinger radial equation with the use of phenomenological Woods-Saxon central potential including spin-orbit interaction. According to our results, the calculated shell levels for neutron single particle in ^{118}Sn are $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$, $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$. Table (1) is the calculated results for neutron single-particle energy levels in ^{118}Sn. The systematic energy level diagrams for neutron is displayed in Fig. (1). The corresponding wave functions for various states are described in Fig. (2), (3) and (4) respectively.

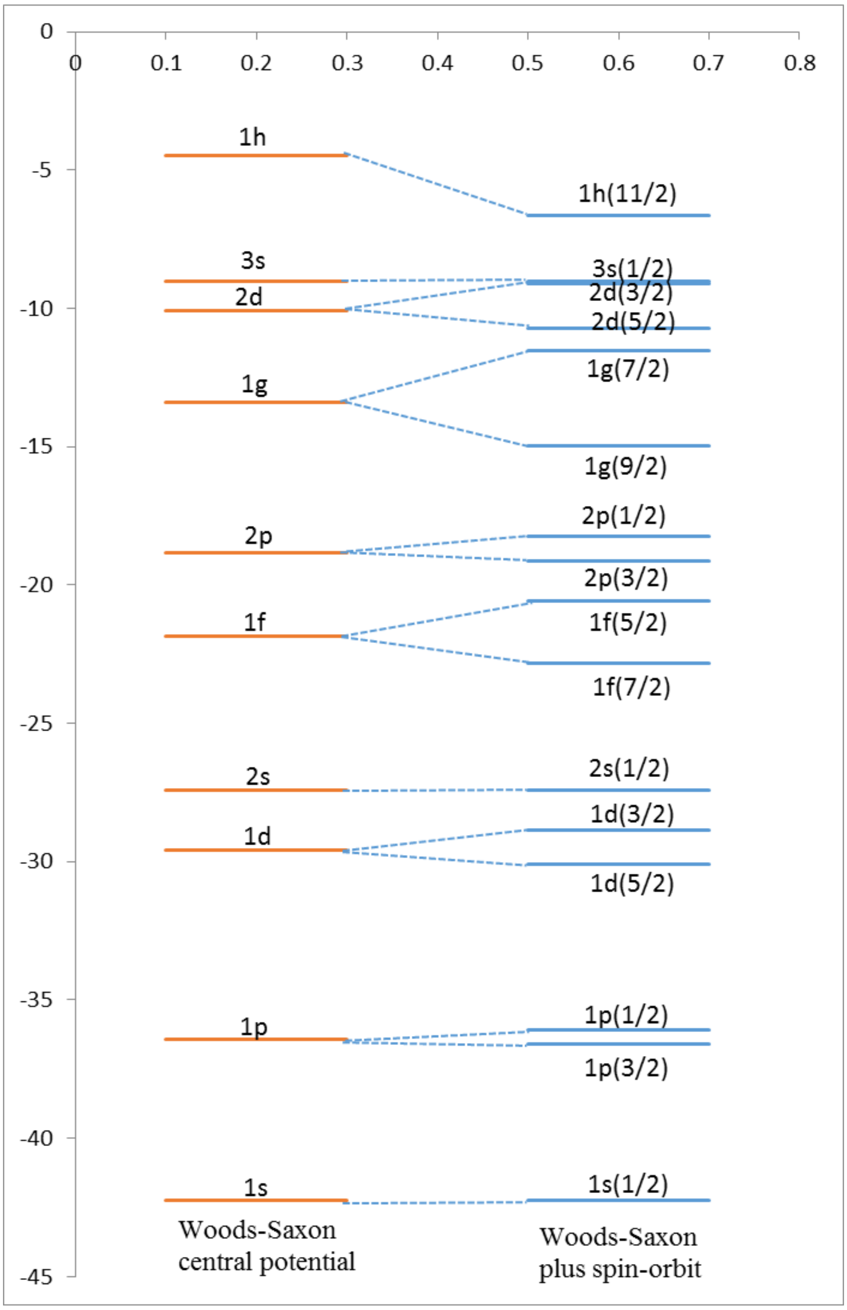

Figure 1 Neutron single-particle energy levels in ^{118}Sn

According to our results, it is found that the attractive interaction strength for the higher orbital angular momentum in each particle state is weaker. It is also known that the attractive interaction strength which possess the total spin J=l+s is stronger than that having J=l-s. We also found that the wave functions of higher orbital angular momentum states shift to the horizontally right. Thus it is also seen that the single-neutron wave functions for J=l+s are more shifted to the outer region than that having spin state J=l-s. Therefore, it can be said that the grater angular momentum, the probability of find the nuclear bound state becomes smaller. From this calculation, we can know the last neutron state which can be bound in the nucleus. In ^{118}Sn, nuclear bound states may exist in $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$, $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ .So it can be said that no neutron can exist upper $1h_{11/2}$ state in ^{118}Sn nucleus.

Table 1 Neutron single particle energy levels by Woods-Saxon potential

Single-particle state	**Single-particle energy(MeV) (central term)**	**Single-particle state**	**Single-particle energy(MeV) (central term+ L-S coupling term)**	**Experimental Results(1)**
1s	-42.258	$1s_{1/2}$	-42.26	
1p	-36.4415	$1p_{3/2}$	-36.61	
		$1p_{1/2}$	-36.098	
1d	-29.576	$1d_{5/2}$	-30.08	
		$1d_{3/2}$	-28.848	
2s	-27.4145	$2s_{1/2}$	-27.4145	
1f	-21.845	$1f_{7/2}$	-22.81	
		$1f_{5/2}$	-20.598	
2p	-18.823	$2p_{3/2}$	-19.12	
		$2p_{1/2}$	-18.235	
1g	-13.415	$1g_{9/2}$	-14.969	
		$1g_{7/2}$	-11.539	9.92±1
2d	-10.09	$2d_{5/2}$	-10.736	9.61±0.96
		$2d_{3/2}$	-9.11	7.57±0.80
3s	-9.026	$3s_{1/2}$	-9.026	8.39±0.84
1h	-4.4665	$1h_{11/2}$	-6.664	7.13±0.71

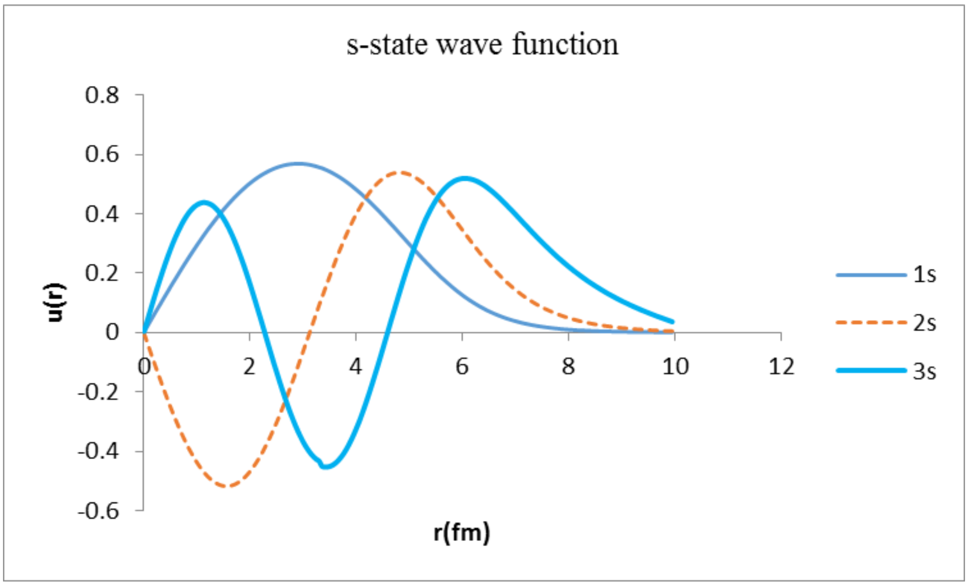

Figure 2 Neutron s-states wave function with Woods-Saxon spin-orbit potential for^{118}Sn

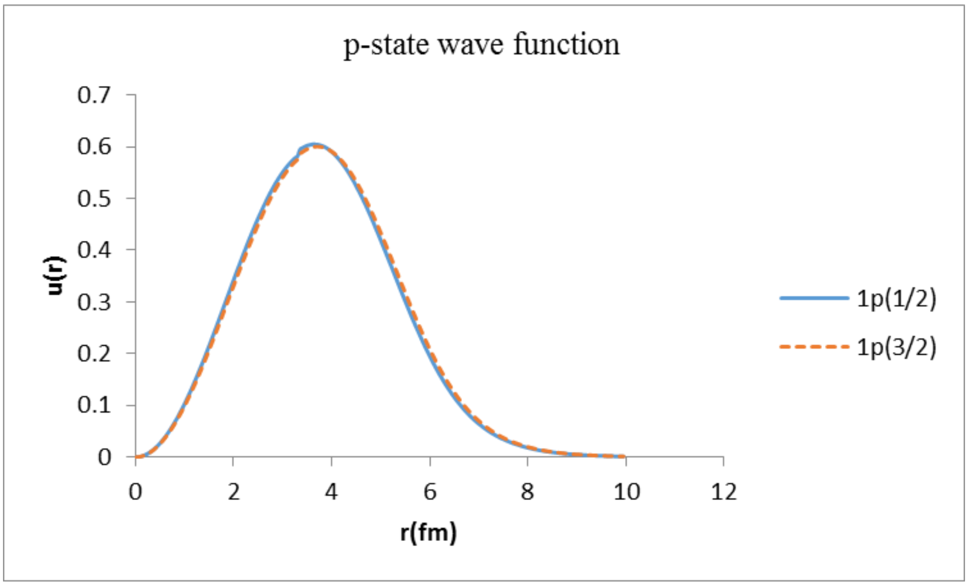

Figure 3 Neutron p-states wave function with Woods-Saxon spin-orbit potential for^{118}Sn

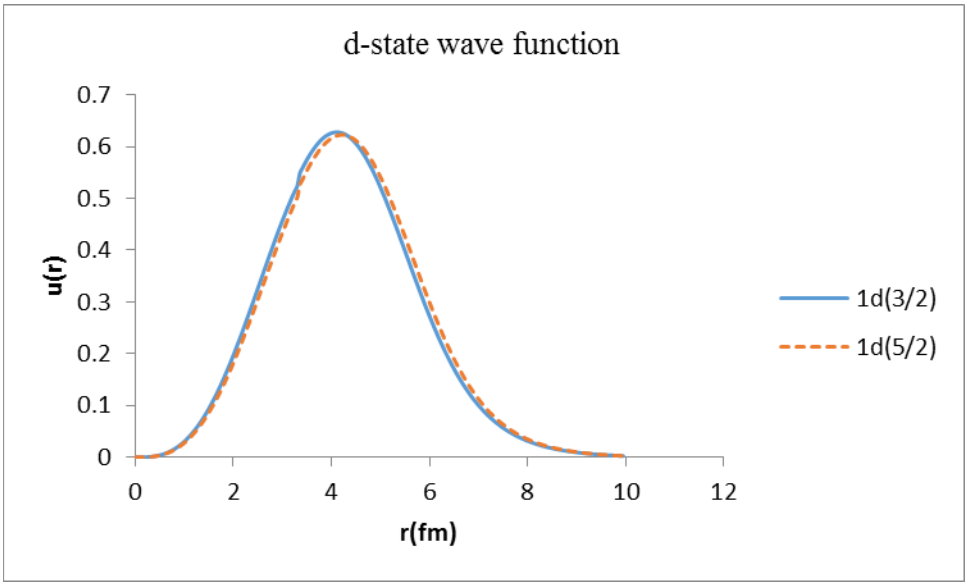

Figure 4 Neutron d-states wave function with Woods-Saxon spin-orbit potential for ^{118}Sn

Conclusion

Our calculated neutron single particle energy of ^{118}Sn are a little smaller than the experimental results (1). One reason of this case is that the imaginary potential that was taken from the systematic of global parameter of conventional optical model potential and the force parameters of its volume and surface parts are neglected in our calculation. For large even-even nuclei, the paring interaction in nuclei for different subshells and the core excitation potential should be taken into account in the interaction term. If we would consider this system by including these factors in the interaction, it will be suggested that more accurate value of the neutron single particle energy of ^{118}Sn which would be fit to the experimental data.

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