ESTIMATION OF RESONANCE POSITION OF THE ${}^{12}_{\Sigma}\mathrm{C}$ HYPERNUCLEAR SYSTEM WITH COMPLEX SQUARE WELL POTENTIAL BY USING GREEN'S FUNCTION METHOD

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Abstract

In this research, differential cross section of ${}^{12}_{6}C(\overline{K},\pi)^{12}_{\Sigma}C$ reaction is formulated by Green's function method. Imaginary part of this formulation, called spectral function, gives the spectral shape of this reaction. From the spectrum, the resonance position of the reaction is investigated. Green's function for this reaction is numerically solved. Then the spectral function is calculated and the various spectral shapes are investigated with simple case of complex square-well potentials. The resulting spectral shapes are compared with those of Morimatsu and Yazaki, obtained by analytically solved Green's function. It was observed that the spectral shapes obtained by both methods, i.e., numerical and analytical, are almost identical. This fact shows an advantage of numerical method in cases where analytical solutions are not possible.

Keywords: *Green's function, spectral function, resonance position.*

Introduction

In 1984, Morimatsu and Yazaki first gave a formalism for treating unstable (or continuum) states in the formation processes. It had been shown that the formation probabilities can be calculated from the *analytically solved Green's function*. Then they used this formalism to examine the effect of the unstable bound state on the formation probabilities. The simple case of a complex square-well potential is studied in detail. They showed that the unstable bound state gives no observable effect either in the scattering or in the formation process.

Many physical quantities are varying functions. For example, formation cross section is slowly varied with the total energy of the system. But at certain energy, the value of cross section is rapidly varied and spectral shape becomes with a very high peak. This phenomenon is called resonance. Then the energy value and observed level width of this high peak will give the resonance position of the reaction.

The purpose of our research is to study the spectral shape of ${}^{12}_{6}\mathrm{C}(\overline{\mathrm{K}},\pi)^{12}_{\Sigma}\mathrm{C}$ reaction with complex square-well potential by using *numerically solved Green's function*. Then we will compare these spectral shapes with those treated by analytically solved Green's function which is proposed by Morimatsu and Yazaki. The numerically solved Green function and the calculations of formation probability of the reaction are given in this research.

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Formulation of Formation Cross Section of ${}^{12}_{6}\mathrm{C}(\overline{\mathrm{K}},\pi)^{12}_{\Sigma}\mathrm{C}$

Transition Matrix Element

The transition matrix element can be generally expressed as

$$\mathrm{T_{fi}} = \langle \text{Final State} | \mathrm{T} | \text{ Initial State} \rangle . \qquad (1)$$

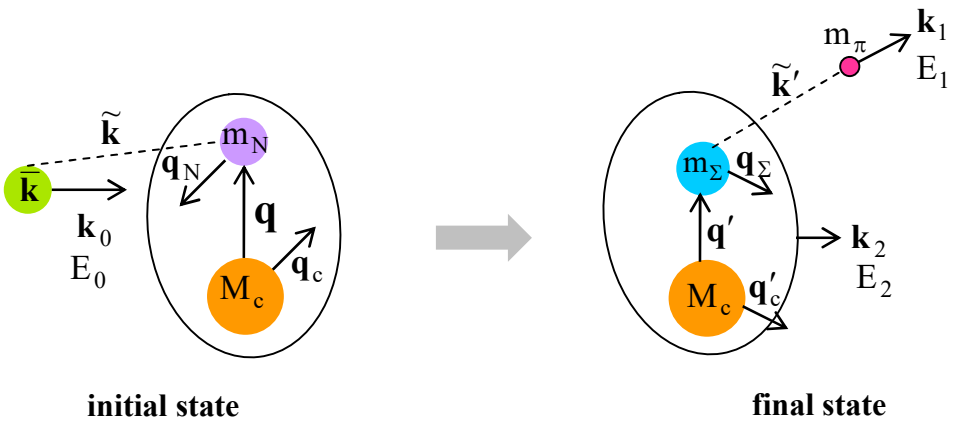


Figure 1 Schematic diagram of the ${}^{12}_{6}\mathrm{C}(\overline{\mathrm{K}},\pi)^{12}_{\Sigma}\mathrm{C}$ reaction

In the initial state of the reaction, $\mathbf{k}_0$ and E_0 are momentum and total energy of incident kaon. $\mathbf{q}_N$ and $\mathbf{q}_c$ are internal momentum of a nucleon and core nucleus. $\mathbf{q}$ is the internal momentum between nucleon and core nucleus. $\tilde{\mathbf{k}}$ is relative momentum between incident kaon and one nucleon of target nucleus. Then in the final state of the reaction, $\mathbf{k}_1$ and E_1 are momentum and total energy of outgoing pion. $\mathbf{k}_2$ and E_2 are momentum and total energy of recoil nucleus. $\mathbf{q}_\Sigma$ and $\mathbf{q}'_c$ are internal momentum of Σ-hypernucleon and core nucleus. $\mathbf{q}'$ is internal momentum between Σ-hypernucleon and core nucleus. $\tilde{\mathbf{k}}'$ is relative momentum between outgoing pion and Σ-hypernucleon.

Transition matrix element $\mathrm{T_{fi}}$ for the above reaction can be expressed in terms of relative and center of mass momenta of initial state and final state as

$$T_{fi}^{(n)} = \frac{\hbar c}{\sqrt{2E_1}} \left[\mathbf{k}_1, \mathbf{k}_2, \Psi_f^{(n)} \right| T \left| \mathbf{k}_0, 0, \Psi_i \right] \frac{\hbar c}{\sqrt{2E_0}}, \qquad (2)$$

$\frac{\hbar c}{\sqrt{2E_0}}$ and $\frac{\hbar c}{\sqrt{2E_1}}$ are normalization constants for incoming kaon wave function and outgoing pion wave function. In order to account the all effects of all internal momenta of initial state and final state, completeness relations corresponding with these momenta are inserted into the equation of transition matrix element as

$$\frac{\sqrt{2E_0}\sqrt{2E_1}}{(\hbar c)^2}T_{fi}^{(n)} = (\tfrac{L}{2\pi})^{3\times4}\iiiint d\mathrm{q}_N\, d\mathrm{q}_c\, d\mathrm{q}_\Sigma d\mathrm{q}'_c \left[\Psi_f^{(n)}, \mathrm{k}_2 \middle| \mathrm{q}_\Sigma, \mathrm{q}'_c\right]$$

$$\left[\ \mathbf{q}_\Sigma, \mathbf{q}'_c, \mathbf{k}_1 \middle| T \middle| \mathbf{k}_0, \mathbf{q}_N, \mathbf{q}_c\ \right]\left[\ \mathbf{q}_N, \mathbf{q}_c\ \middle| 0, \Psi_i\ \right]. \qquad (3)$$

The internal particular momenta of $\mathbf{q}_\Sigma$ and $\mathbf{q}'_c$ can be expressed into relative and center of mass momentum as

$$\frac{\sqrt{2E_0}\sqrt{2E_1}}{(\hbar c)^2}T_{fi}^{(n)} = (\tfrac{L}{2\pi})^{3\times4}\iiiint d\mathbf{q}_N\, d\mathbf{q}_c\, d\mathbf{q}_\Sigma d\mathbf{q}'_c \left[\Psi_f^{(n)} \middle| \mathbf{q}'\right]\left[\mathbf{k}_2 \middle| \mathbf{q}_\Sigma + \mathbf{q}'_c\right]$$

$$\left[\ \mathbf{q}_\Sigma, \mathbf{q}'_c, \mathbf{k}_1 \middle| T \middle| \mathbf{k}_0, \mathbf{q}_N, \mathbf{q}_c\ \right]\left[\ \mathbf{q}_N + \mathbf{q}_c\ \middle| 0\ \right]\left[\ \mathbf{q}\ \middle| \Psi_i\ \right] \qquad (4)$$

The terms, $\left[\mathbf{k}_2 \middle| \mathbf{q}_\Sigma + \mathbf{q}'_c\right]$, $\left[\mathbf{q}_N + \mathbf{q}_c\ \middle| 0\right]$, and $\left[\ \mathbf{q}_\Sigma, \mathbf{q}'_c, \mathbf{k}_1 \middle| T \middle| \mathbf{k}_0, \mathbf{q}_N, \mathbf{q}_c\ \right]$ from above equation are particularly solved. Since, the transition matrix element $\left[\ \tilde{\mathbf{k}}' \middle| T \middle| \tilde{\mathbf{k}}\ \right]$ depends only on the relative momenta of the reaction, we can therefore change the notation of transition operator 'T' by 't'. And then (4) becomes as

$$\frac{\sqrt{2E_0}\sqrt{2E_1}}{(\hbar c)^2}T_{fi} = (\tfrac{L}{2\pi})^{3\times4}\iiiint d\mathbf{q}_N d\mathbf{q}_c d\mathbf{q}_\Sigma d\mathbf{q}'_\mathbf{c} \left\langle \Psi_\mathbf{f}^{(\mathbf{n})} \middle| \mathbf{q}'\right](\tfrac{2\pi}{L})^3\delta(\mathbf{k}_2 - \mathbf{q}_\Sigma - \mathbf{q}'_c)$$

$$\left[\ \tilde{\mathbf{k}}' \middle| t \middle| \tilde{\mathbf{k}}\ \right](\tfrac{2\pi}{L})^3\delta(\mathbf{k}_1 + \mathbf{q}_\Sigma - \mathbf{k}_0 - \mathbf{q}_N)(\tfrac{2\pi}{L})^3\delta(\mathbf{q}'_c - \mathbf{q}_\mathbf{c})(\tfrac{2\pi}{L})^3$$

$$\delta(\mathbf{q}_N + \mathbf{q}_c)\left[\mathbf{q} \middle| \Psi_i\right\rangle. \qquad (5)$$

By using the delta function properties, (5) becomes as

$$T_{fi} = \frac{(\hbar c)^2}{\sqrt{2E_0}\sqrt{2E_1}}\delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_0)\int d\mathbf{q}_c \left\langle \Psi_f^{(n)} \middle| \mathbf{q}'\right]\left[\ \tilde{\mathbf{k}}' \middle| t \middle| \tilde{\mathbf{k}}\ \right]\left[\mathbf{q} \middle| \Psi_i\right\rangle \qquad (6)$$

where $\mathbf{q} = -\mathbf{q}_c$, $\mathbf{q}' = \frac{M_c}{M_c + m_\Sigma}\mathbf{k}_2 - \mathbf{q}_c$, $\tilde{\mathbf{k}} = \frac{m_N}{m_{\bar{k}} + m_N}\mathbf{k}_0 + \frac{m_{\bar{k}}}{m_{\bar{k}} + m_N}\mathbf{q}_c$, $\tilde{\mathbf{k}}' = \frac{m_\Sigma \mathbf{k}_1 - m_\pi \mathbf{k}_2}{m_\pi + m_\Sigma} + \frac{m_\pi \mathbf{q}_\mathbf{c}}{m_\pi + m_\Sigma}$

Transition probability is

$$\left|T_{fi}\right|^2 = \left|\frac{(\hbar c)^2}{\sqrt{2E_0}\sqrt{2E_1}}\delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_0)\int d\mathbf{q}_c \left\langle \Psi_f^{(n)} \middle| \mathbf{q}'\right]\left[\tilde{\mathbf{k}}' \middle| t \middle| \tilde{\mathbf{k}}\right]\left[\mathbf{q} \middle| \Psi_i\right\rangle\right|^2$$

$$= \frac{(\hbar c)^4}{4E_0E_1}\left|\delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_0)\right|^2\left|\int d\mathbf{q}_c \left\langle \Psi_f^{(n)} \middle| \mathbf{q}'\right]\left[\tilde{\mathbf{k}}' \middle| t \middle| \tilde{\mathbf{k}}\right]\left[\mathbf{q} \middle| \Psi_i\right\rangle\right|^2. \qquad (7)$$

Using relations between delta function and box function normalization, $\left|\mathbf{q}'\right] = (\frac{2\pi}{L})^{3/2}\left|\mathbf{q}'\right\rangle$, $\left[\tilde{\mathbf{k}}'\right| = \left\langle\tilde{\mathbf{k}}'\right|(\frac{2\pi}{L})^{3/2}$, $\left|\mathbf{q}\right] = \left\langle\mathbf{q}\right|(\frac{2\pi}{L})^{3/2}$ matrix element is

$$\left|T_{fi}\right|^2 = \frac{(\hbar c)^4}{4E_0E_1}(\tfrac{2\pi}{L})^9\delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_0)\left|\int d\mathbf{q}_c \left\langle \Psi_f^{(n)} \middle| \mathbf{q}'\right\rangle\left\langle\tilde{\mathbf{k}}' \middle| t \middle| \tilde{\mathbf{k}}\right\rangle\left\langle\mathbf{q} \middle| \Psi_i\right\rangle\right|^2. \qquad (8)$$

Differential Cross Section

After the matrix element of the reaction is calculated transition rate and cross section are obtained by following relations:

$$W_{fi} = \tfrac{2\pi}{\hbar}\sum_{n}\left|T_{fi}\right|^2 \delta(E_f - E_i)\rho(E) \tag{9}$$

$$Cross\ Section = \frac{transition\ \ rate}{incident\ \ flux} = \frac{W_{fi}}{v_0/L^3}\,. \tag{10}$$

W_{fi} = transition rate

T_{fi} = transition probability

$\delta(E_f - E_i)$ = energy conservation term

$\rho(E)$ = number of allowed state(or) phase volume

$\frac{v_0}{L^3}$ = incident flux

The value of the transition probability $\left|T_{fi}\right|^2$ is inserted into the above equation (9) as

$$W_{fi} = \frac{2\pi}{\hbar}\frac{(\hbar c)^4}{4E_0E_1}\left(\frac{2\pi}{L}\right)^9\sum_{n}\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_0)\left|\int d\mathbf{q}_c\left\langle\Psi_f^{(n)}\middle|\mathbf{q}'\right\rangle\left\langle\widetilde{\mathbf{k}}'\middle|t\middle|\widetilde{\mathbf{k}}\right\rangle\left\langle\mathbf{q}\middle|\Psi_i\right\rangle\right|^2$$

$$\delta(E_f - E_i)\rho(E)\,. \tag{11}$$

The density of final state, $\rho(E) = \left(\frac{L}{2\pi}\right)^6 dk_1 dk_2$, the number of states per unit energy interval, can be easily calculated by using the periodic boundary condition and the wave functions for final state in Cartesian coordinate.

$$W_{fi} = \tfrac{2\pi}{\hbar}\tfrac{(\hbar c)^4}{4E_0E_1}\left(\tfrac{2\pi}{L}\right)^9\left(\tfrac{L}{2\pi}\right)^6\sum_{n}\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_0)\left|\int d\mathbf{q}_c\left\langle\Psi_f^{(n)}\middle|\mathbf{q}'\right\rangle\left\langle\widetilde{\mathbf{k}}'\middle|t\middle|\widetilde{\mathbf{k}}\right\rangle\left\langle\mathbf{q}\middle|\Psi_i\right\rangle\right|^2$$

$$\delta(E_f - E_i)d\mathbf{k}_1 d\mathbf{k}_2 \tag{12}$$

The differential cross section can be written as

$$d^6\sigma = \tfrac{L^3}{v_0}W_{fi}$$

$$= \tfrac{2\pi}{\hbar}\tfrac{(\hbar c)^4}{4E_0E_1}\left(\tfrac{2\pi}{L}\right)^9\left(\tfrac{L}{2\pi}\right)^6\tfrac{L^3E_0}{c^2\hbar k_0}\sum_{n}\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_0)\left|\int d\mathbf{q}_c\left\langle\Psi_f^{(n)}\middle|\mathbf{q}'\right\rangle\left\langle\widetilde{\mathbf{k}}'\middle|t\middle|\widetilde{\mathbf{k}}\right\rangle\left\langle\mathbf{q}\middle|\Psi_i\right\rangle\right|^2$$

$$\delta(E_f - E_i)d\mathbf{k}_1 d\mathbf{k}_2 \tag{13}$$

where $v_0 = \frac{c^2\hbar k_0}{E_0}$. After calculation the integral term $\int d\mathbf{k}_2$, the differential cross section is obtained.

$$d^3\sigma = (2\pi)^4\frac{(\hbar c)^2}{4k_0E_1}\sum_{n}\left|\int d\mathbf{q}_c\left\langle\Psi_f^{(n)}\middle|\mathbf{q}'\right\rangle\left\langle\widetilde{\mathbf{k}}'\middle|t\middle|\widetilde{\mathbf{k}}\right\rangle\left\langle\mathbf{q}\middle|\Psi_i\right\rangle\right|^2\delta(E_f - E_i)d\mathbf{k}_1 \tag{14}$$

Energy Conservation Term

Then energy conservation term $\delta(E_i - E_f)$ can be written by using delta function properties, $\lim_{\varepsilon\to 0}\frac{1}{x+i\varepsilon} = \frac{P}{x} - i\pi\delta(x)$ as

$$\delta(E_i - E_f) = \left\langle \Psi_f^{(n)} \left| \delta(E - H_{\Sigma c}) \right| \Psi_f^{(n)} \right\rangle \tag{15}$$

$$\delta(\mathrm{E_i} - \mathrm{E_f}) = \left(-\tfrac{1}{\pi}\right)\mathrm{Im}\left\langle \Psi_{\mathrm{f}}^{(\mathrm{n})} \left| \frac{1}{\mathrm{E}-\mathrm{H}_{\Sigma \mathrm{c}}+\mathrm{i}\varepsilon} \right| \Psi_{\mathrm{f}}^{(\mathrm{n})} \right\rangle . \tag{16}$$

Differential cross section equation (14) becomes as

$$d^3\sigma = (2\pi)^4 \frac{(\hbar c)^2}{4k_0 E_1} \sum_n \left| \int d\mathbf{q}_c \left\langle \Psi_f^{(n)} \middle| \mathbf{q}' \right\rangle \left\langle \tilde{\mathbf{k}}' \middle| t \middle| \tilde{\mathbf{k}} \right\rangle \left\langle \mathbf{q} \middle| \Psi_i \right\rangle \right|^2 \left(-\tfrac{1}{\pi}\right)\mathrm{Im}\left\langle \Psi_f^{(n)} \left| \frac{1}{E-H_{\Sigma c}+i\varepsilon} \right| \Psi_f^{(n)} \right\rangle d\mathbf{k}_1 . \tag{17}$$

The differential cross section can be written in terms of coordinate representation as

$$d^3\sigma = (2\pi)^4 \frac{(\hbar c)^2}{4k_0 E_1} \left|\langle \bar{t} \rangle\right|^2 \left(-\tfrac{1}{\pi}\right) \mathrm{Im}\left[\iint d\mathbf{r} d\mathbf{r}' \Psi_i^*(\mathbf{r}) e^{-i\eta(\mathbf{k}_0-\mathbf{k}_1).\mathbf{r}} \left\langle \mathbf{r} \left| \frac{1}{E-H_{\Sigma c}+i\varepsilon} \right| \mathbf{r}' \right\rangle \right.$$

$$\left. e^{i\eta(\mathbf{k}_0-\mathbf{k}_1).\mathbf{r}'} \Psi_i(\mathbf{r}') \right] d\mathbf{k}_1 . \tag{18}$$

The differential cross section per unit solid angle of outgoing pion for the reaction ${}^{12}_{6}C(\overline{K},\pi)^{12}_{\Sigma}C$ as

$$\frac{d^3\sigma}{dE_1 d^2\Omega_1} = (2\pi)^4 \frac{(\hbar c)^2 k_1}{4k_0 \hbar^2 c^2} \left|\langle \bar{t} \rangle\right|^2 \left(-\tfrac{1}{\pi}\right) \mathrm{Im}\left[\iint d\mathbf{r} d\mathbf{r}' f^*(\mathbf{r}) \left\langle \mathbf{r} \left| \frac{1}{E-H_{\Sigma c}+i\varepsilon} \right| \mathbf{r}' \right\rangle f(\mathbf{r}')\right]. \tag{19}$$

From the above equation, the imaginary term can give the spectral shape of the differential cross section with respect to the total energy of the outgoing pion. Before we calculate the spectral function, we first evaluate the matrix element, $\left\langle \mathbf{r} \left| \frac{1}{E-H_{\Sigma c}+i\varepsilon} \right| \mathbf{r}' \right\rangle$, by using Green's function method.

Green's Function and Spectral Function

Formulation of Green's Function

The matrix element $\left\langle \mathbf{r} \left| \frac{1}{E-H_{\Sigma c}+i\varepsilon} \right| \mathbf{r}' \right\rangle$ can be defined as Green's function $G(\mathbf{r},\mathbf{r}')$. Green's function can be separated into radial part and angular part. The Green's function $G(\mathbf{r},\mathbf{r}')$ can be separated into radial part and angular part as

$$\mathrm{G}(\mathbf{r},\mathbf{r}') = \sum_{\ell=0}^{\infty} \sum_{\mathrm{M}} \mathrm{Y}_{\ell \mathrm{M}}(\hat{\mathbf{r}}) \frac{\mathrm{G}(\mathrm{r},\mathrm{r}')}{\mathrm{r}\,\mathrm{r}'} \mathrm{Y}^*_{\ell \mathrm{M}}(\hat{\mathbf{r}}') . \tag{20}$$

The radial part of Green's function satisfies the Schrödinger radial equation

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{\ell(\ell+1)}{r^2} - \tilde{V}(r)\right] G(r,r') = \frac{2\mu}{\hbar^2}\delta(r,r') \tag{21}$$

where $k = \sqrt{\frac{2\mu E}{\hbar^2}}$ and $\widetilde{V}(r) = \frac{2\mu}{\hbar^2} v(r)$. The Green's function is continuous and its derivative has discontinuity. It is convenient to consider the two intervals a ≤ r < r′ and r′ < r ≤ b separately and write the Green's function in the following forms.

$$G(r,r') = G_1(r,r') \quad a \le r < r' \tag{22}$$

$$G(r,r') = G_2(r,r') \quad r' < r \le b \tag{23}$$

By applying the properties of the Green's function: the Green's function is continuous at r = r′; and the derivative of the Green's function has discontinuity at r = r′, the two solutions G_1 (r, r′) and G_2 (r, r′) become as

$$G_1(r,r') = \frac{2\mu}{\hbar^2} \frac{v_\ell(r') u_\ell(r)}{W(u_\ell, v_\ell)} \qquad 0 \le r < r' \tag{24}$$

and

$$G_2(r,r') = \frac{2\mu}{\hbar^2} \frac{u_\ell(r') v_\ell(r)}{W(u_\ell, v_\ell)} \qquad r' < r \le \infty \tag{25}$$

where $W = u_\ell(r')v'_\ell(r') - u'_\ell(r')v_\ell(r')$ is Wronskian of u(r′) and v(r′) and will be non-zero if u(r′) and v(r′) are linearly independent. The values of r′ in Eq. (24) are greater than r and the values of r in Eq. (25) are greater than r′. Then we defined as $\mathrm{v}_\ell(\mathrm{r}_>)$. Similarly, we defined for $\mathrm{u}_\ell(\mathrm{r}')$ and $\mathrm{u}_\ell(\mathrm{r})$ as $\mathrm{u}_\ell(\mathrm{r}_<)$ where $\mathrm{r}_<$ and $\mathrm{r}_>$ are the larger and the smaller values of 'r' and 'r'' respectively. These two solutions are recombined into G(r, r′) as

$$G(r,r') = \frac{2\mu}{\hbar^2} \frac{u_\ell(r_<) v_\ell(r_>)}{W(u_\ell, v_\ell)} \tag{26}$$

By inserting the value of G(r, r′) from (26) into (20), we get

$$G(\mathbf{r},\mathbf{r}') = \frac{2\mu}{\hbar^2} \sum_{\ell=0}^{\infty} \sum_{M} Y_{\ell M}(\hat{\mathbf{r}}) \frac{1}{rr'} \frac{u_\ell(r_<) v_\ell(r_>)}{W(u_\ell, v_\ell)} Y^*_{\ell M}(\hat{\mathbf{r}}') \tag{27}$$

where $\mathrm{u}_\ell(\mathrm{r}')$ and $\mathrm{v}_\ell(\mathrm{r}')$ are the stationary and out-going solutions of the Schrödinger equation. The boundary conditions of these solutions are $\mathrm{u}_\ell(0) = \mathrm{r}^{\ell+1}$ at the origin and $\mathrm{v}_\ell(\mathrm{r}) = \mathrm{kr}\,\mathrm{h}^+_\ell(\mathrm{kr})$ in the asymptotic region $\mathrm{r} \to \infty$, respectively. These particular solutions can be derived by solving Schrödinger radial equation with the use of difference method.

Calculation of Spectral Function

In order to study the spectral shape of the reaction, we considered only the spectral function from the formulation of formation cross section as

$$S(E) = (-\tfrac{1}{\pi}) \mathrm{Im}\Big[\iint d\mathbf{r} d\mathbf{r}' \Psi^*_i(\mathbf{r}) e^{-i\boldsymbol{\eta}(\mathbf{k}_0 - \mathbf{k}_1).\mathbf{r}} \left\langle \mathbf{r} \left| \tfrac{1}{E - H_{\sum c} + i\boldsymbol{\varepsilon}} \right| \mathbf{r}' \right\rangle$$
$$e^{i\boldsymbol{\eta}(\mathbf{k}_0 - \mathbf{k}_1).\mathbf{r}'} \Psi_i(\mathbf{r}') \Big] \tag{28}$$

The simple notation '**Q**' is used for $\eta(\mathbf{k}_0 - \mathbf{k}_1)$ and mathematical expansion of the free particle state wave function as

$$e^{i\mathbf{Q}.\mathbf{r}} = \sum_{\ell=0}^{\infty} \sqrt{4\pi(2\ell+1)}\; i^{\ell} j_{\ell}(Qr') Y_{\ell \mathbf{0}}(\hat{\mathbf{r}}') \quad \text{and} \tag{29}$$

$$e^{-i\mathbf{Q}.\mathbf{r}} = \sum_{\ell=0}^{\infty} \sqrt{4\pi(2\ell+1)}\,(-i)^{\ell} j_{\ell}(Qr) Y^{*}_{\ell 0}(\hat{\mathbf{r}}). \tag{30}$$

The solution of Green's function is

$$\left\langle \mathbf{r} \middle| \frac{1}{E - H_{\Sigma c} + i\boldsymbol{\varepsilon}} \middle| \mathbf{r}' \right\rangle = \frac{2\mu}{\hbar^2} \sum_{\ell=0}^{\infty} \sum_{M} Y_{\ell M}(\hat{\mathbf{r}}) \frac{1}{rr'} \frac{u_{\ell}(r_<) v_{\ell}(r_>)}{W(u_{\ell}, v_{\ell})} Y^{*}_{\ell M}(\hat{\mathbf{r}}'). \tag{31}$$

The imaginary term from Eq. (28) will give spectral shape of this reaction. By substituting the solution of Green's function Eq. (31), harmonic oscillator wave function of carbon target for s-state and mathematical expansions of free particle wave functions, we have solved the imaginary term called spectral function. Since the solutions of $\mathrm{u}_{\ell}(\mathrm{r}_<)$ and $\mathrm{v}_{\ell}(\mathrm{r}_>)$ depend on the total energy of Σ-core-nucleus, the spectral function is the function of $\mathrm{E}_{\Sigma\text{-core}}$. Then we obtained the spectral function as

$$\mathrm{S(E)} = -4\,\frac{2\mu}{\hbar^2}\left(\tfrac{1}{a^2\boldsymbol{\pi}}\right)^{3/2} \mathrm{Im}\left[\iint r\,r'\,\exp\left(\tfrac{-r^2}{2a^2}\right)\exp\left(\tfrac{-r'^2}{2a^2}\right)\sum_{\ell=0}^{\infty}(2\ell+1) j_{\ell}(Qr)\, j_{\ell}(Qr')\right.$$

$$\left.\frac{u_{\ell}(r_<) v_{\ell}(r_>)}{W(u_{\ell}, v_{\ell})}\, dr\, dr'\right]. \tag{32}$$

The spectral functions for various total energies of Σ-core-nucleus are numerically calculated and compared with those calculated by Morimatsu and Yazaki. The obtaining results are explicitly discussed.

Result and Discussion

In order to simulate the spectral shapes for the ^{12}C target, we used two cases of complex square-well potentials from as shown in Table (1). Our calculated spectral functions with arbitrary unit for above two cases are compared with those of Morimatsu and Yazaki. It was found that all results are equivalent.

For case (a) potential, (-26, -2*i*) MeV, the spectral function calculated with numerically solved Green's function (i.e., our result) is shown in Fig. (2) and that with analytically solved Green's function calculated by Morimatsu and Yazaki is also shown in Fig. (3). The observed peaks are found at about -3.47MeV (our result) and -3.48 MeV (result of Morimatsu and Yazaki) of $\mathrm{E}_{\Sigma\text{-core}}$ respectively. The two results are agreed.

The spectral functions for case (b) potential, (-16, 0*i*) MeV, are expressed in Fig. (4) and Fig. (5) for our result and their result respectively. The sharp peaks are found at 1.38 MeV (our result) and at 1.16 MeV (their result). For this case, we also determined the spectral functions with various values of imaginary part of potentials, W= -2, -3 and -6 MeV. These are shown in Fig. (6). In this figure, it is observed that as |W| increases, the peak positions of these cases are dramatically decreases. Since the results for various |W| from (Morimatsu and Yazaki) are also

shown in Fig. (7), our results can be clearly compared with those. It can be observed that the two results are almost identical.

Table 1 The strengths of the complex square-well potentials (Morimatsu and Yazaki)

Case	V(MeV)	W(MeV)
a	-26	-2
b	-16	0

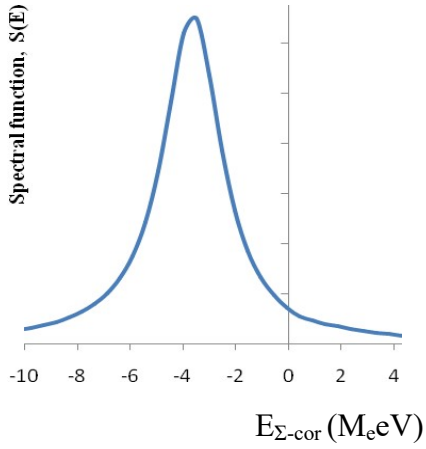


Figure 2 Spectral function calculated by numerically solved Green's funciton for case (a) (our result).

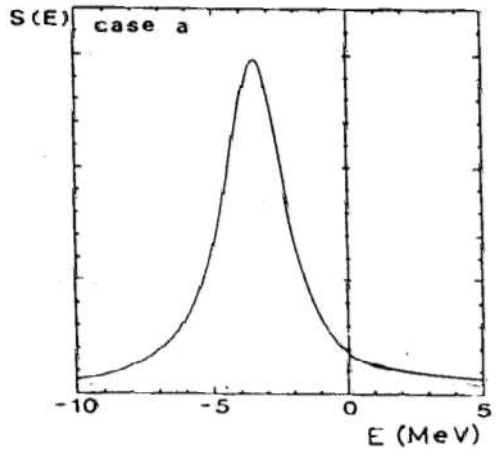


Figure 3 Spectral function calculated by analytically solved Green's funciton for case (a) (Result of Morimatsu and Yazaki)

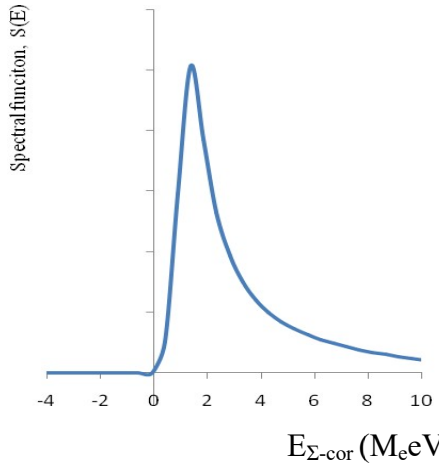


Figure 4 Spectral function calculated by numerically solved Green's funciton for case (b) (our result).

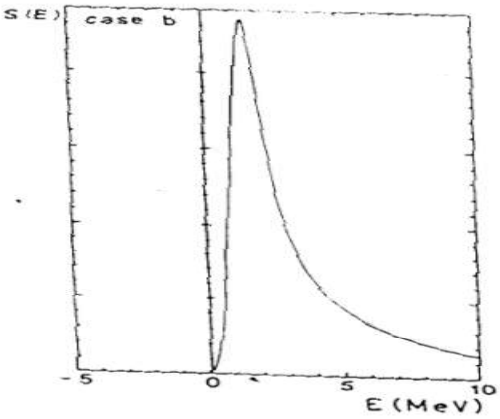


Figure 5 Spectral function calculated by analytically solved Green's funciton for case (b) (Result of Morimatsu and Yazaki)

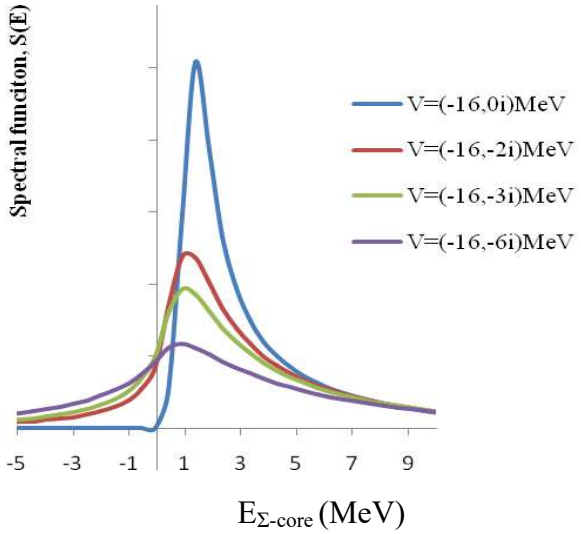
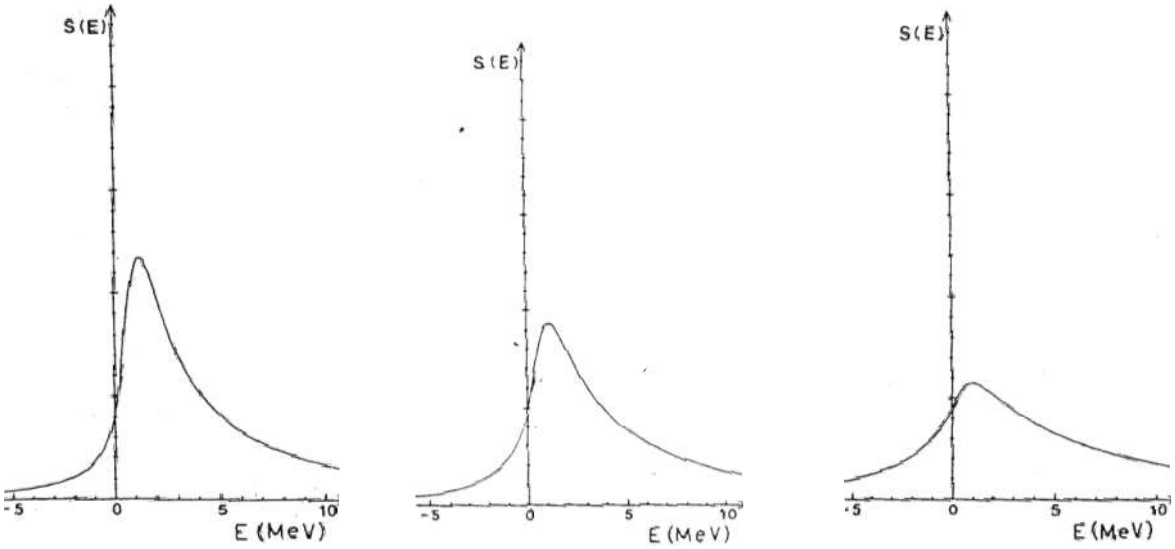


Figure 6 Spectral functions calculated by numerically solved Green's funciton for case (b) with W=0, -2, -3 and -6 MeV (our results).



(W=-2 MeV) (W=-3 MeV) (W=-6 MeV)

Figure 7 Spectral functions calculated by analytically solved Green's funciton for case (b) with W= -2, -3 and -6 MeV (Results of Morimatsu and Yazaki).

Conclusion

We firstly concluded that formation cross section can be calculated by the use of Green's function method. Secondly, the solutions of Green's function can be obtained by both numerically and analytically. Since these solutions are equivalent, numerically solved Green's function can also be used to study the formation cross section in nuclear physics. Thirdly, since the spectral function can give the resonance position of the Σ-hypernuclear system, we can estimate the total energy or binding energy of the Σ-hypernuclear state.

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