INVESTIGATION ON PROTON AND LEPTON CONCENTRATION IN SUPERNOVA EXPLOSIONS SIMULATIONS

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Abstract

This research describes the effective chiral mean field model to construct the relativistic equation of state (EOS) of nuclear matter at finite temperature and density with various proton fractions and lepton fractions for the use in the supernova simulations. This model is based on chiral symmetry and we have adopted the RMF theory with the non-linear $\sigma - \omega$ terms, which was a success in describing the properties of both stable and unstable nuclei. In supernovae, the EOS determines the non-linear the dynamics of the collapse and the outgoing shock, and determines whether the remnant ends up as a neutron star or black hole. The various properties of supernova matter are investigated within the effective chiral mean field model. The outcome is demonstrated at zero entropy (neglecting thermal effect), the various proton fractions ranging from (Y_p=0.1~0.5) and various lepton fractions ranging from (Y_e=0.13~0.5) to be used for the simulations.

Keywords: supernova explosions, equation of state, effective chiral mean field model

Introduction

The equation of state (EOS) of nuclear matter is important in various astrophysical phenomena such as supernova explosions and the formation of neutron stars and black holes. The EOS in nuclear astrophysics purposes depends on a series of thermodynamic properties which are obtained for certain temperatures, densities and matter composition. The relativistic equation of state (EOS) of nuclear matter is designed for the use of supernova simulations in the wide density and temperature range with various proton fractions. We have adopted the RMF theory with the non-linear $\sigma - \omega$ terms, which was a success in describing the properties of both stable and unstable nuclei. The various properties of supernova matter are investigated within the effective chiral mean field model. The outcome is demonstrated at zero entropy (neglecting thermal effect), the various proton fractions ranging from (Y_p=0.1~0.5) and various lepton fractions ranging from (Y_e=0.13~0.5) to be used for the simulations in this article.

Chirality and Chiral Symmetry

Chiral comes from the Greek for "Hand". An object that cannot be superimposed on its mirror image is called chiral. Most generally chirality means: the structural characteristic of a finite system (molecule, atom and particle) that makes it impossible to superimpose it on its mirror image. The most straightforward example of chiral symmetry is the mirror symmetry shown by your left and right hand. In quantum field theory, chiral symmetry is a possible symmetry of the Lagrangian under which the left-handed and right-handed parts of Dirac fields transform independently. The chiral symmetry transformation can be divided into a component that treats the left-handed and the right-handed parts equally, known as vector symmetry, and a component that actually treats them differently, known as axial symmetry.

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Equation of State in the Chiral Su (3) Model

The model has been found to describe the hadronic masses of the various SU(3) multiplets, finite nuclei, hypernuclei and excited nuclear matter reasonably well. It consists of a chiral $SU(3)_L \times SU(3)_R$ $\sigma - \omega$ type model. This means that its most important feature is allowing a chirally symmetric phase. The relevant degrees of freedom are baryons that interact through mesons. The scalar mesons (as the σ) represent the attractive part of the strong force while the vector mesons (as the ω) generate the repulsive part. The basic assumptions in the present chiral model are (1) the Lagrangian is constructed with respect to the nonlinear realization of chiral $SU(3)_L \times SU(3)_R$ symmetry; (2) the masses of the heavy baryons and mesons are generated by spontaneous symmetry breaking; (3) the masses of the pseudoscalar mesons are generated by explicit symmetry breaking; (4) a QCD-motivated field χ enters, which describe gluon condensate and baryons and mesons are grouped according to their quark structures.

The total Lagiangian of the chiral $SU(3)_L \times SU(3)_R$ model can be written as

$$\mathcal{L} = \mathcal{L}_{Kin} + \mathcal{L}_{int} + \mathcal{L}_{scal} + \mathcal{L}_{vec} + \mathcal{L}_{SB}$$

where , $\mathcal{L}_{Kin}$ = the kinetic energy term

$\mathcal{L}_{int}$ = the interaction term between baryons and mesons

$\mathcal{L}_{scal}$ = the self-interaction term for the spin-0 mesons

$\mathcal{L}_{vec}$ = the self-interaction term for the spin-1 mesons

$\mathcal{L}_{SB}$ = the explicit symmetry breaking term

Each of these Lagrangian parts will be explained in the following.

$$\mathcal{L}_{Kin} = iTr\left(\overline{B}\gamma_\mu D^\mu B\right) + \frac{1}{2}Tr\left(D_\mu X D^\mu X\right) + Tr\left(\mu_\mu X \mu^\mu X + X \mu_\mu \mu^\mu X\right)$$

$$+\frac{1}{2}Tr\left(D_\mu Y D^\mu Y\right) + \frac{1}{2}Tr\left(D_\mu \chi D^\mu \chi\right) - \frac{1}{4}Tr\left(\tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu}\right) - \frac{1}{4}Tr(\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu})$$

$$\mathcal{L}_{int} = -\sqrt{2}g_8^w\left\{\alpha_w\left[\overline{B}\mathcal{O}BW\right]_{AS} + (1-\alpha_w)\left[\overline{B}\mathcal{O}BW\right]_S\right\} - g_1^w\frac{1}{\sqrt{3}}Tr\left(\overline{B}\mathcal{O}B\right)Tr(W)$$

$$\mathcal{L}_{scal} = \mathcal{L}_{scal} - k_4\chi^4 - \frac{1}{4}\chi^4 ln\frac{\chi^4}{\chi_0^4} + \frac{\delta}{3}\chi^4 ln\frac{I_3}{det\langle X\rangle_0}$$

$$\mathcal{L}_{vec} = \frac{1}{2}m_v^2\frac{\chi^2}{\chi_0^2}Tr\left(V_\mu V^\mu\right) + 2g_4^4 Tr(V_\mu V^\mu)^2$$

$$\mathcal{L}_{SB} = -\frac{\chi^2}{\chi_0^2}Tr(f\Sigma)$$

$$= -\frac{\chi^2}{\chi_0^2}[m_\pi^2 f_\pi \sigma + \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\ ]$$

To apply the effective chiral model to the description of equation of state for supernova matter, we perform the mean-field approximation. The mesons are treated as classical fields, i.e., they are replaced by their expectations values, which are classical fields. Furthermore, if rotational invariance holds, the expectation value of the three vector part of the vector mesons vanishes.

$$\sigma(x) = \langle\sigma\rangle + \delta\sigma \rightarrow \langle\sigma\rangle \equiv \sigma;$$

$$\zeta(x) = \langle\zeta\rangle + \delta\zeta \rightarrow \langle\zeta\rangle \equiv \zeta,$$

$$\omega_\mu(x) = \langle\omega\rangle\delta_{0\mu} + \delta\omega_\mu \rightarrow \langle\omega_0\rangle \equiv \omega;$$

$$\phi_\mu(x) = \langle\phi\rangle\delta_{0\mu} + \delta\phi_\mu \rightarrow \langle\phi_0\rangle \equiv \phi.$$

The total Lagrangian density in the mean field approximation can be written as

$$\mathcal{L}_{MFT} = \mathcal{L}_{Kin} + \mathcal{L}_{int} + \mathcal{L}_{scal} + \mathcal{L}_{vec} + \mathcal{L}_{SB}$$

where, $\mathcal{L}_{Kin} = -\bar{\psi}\gamma_i\partial^\mu\psi - \frac{1}{2}\sum_{\varphi=\sigma,\zeta,\chi,\omega,\rho}\partial_\mu\varphi\partial^\mu\phi,$

$$\mathcal{L}_{int} = -\sum_i \bar{\psi}_i\,(m_i^* + g_{i\omega}\gamma_0\omega^0 + g_{i\phi}\gamma_0\phi^0 + g_{Np}\gamma_0\tau_3\rho_0)\psi_i$$

$$\mathcal{L}_{scal} = -\frac{1}{2}k_0\chi^2(\sigma^2+\zeta^2) + k_1(\sigma^2+\zeta^2)^2 + k_2\left(\frac{\sigma^4}{2}+\zeta^4\right) + k_3\chi\sigma^2\zeta + k_{3m}\chi\left(\frac{\sigma^3}{\sqrt{2}}+\zeta^3\right) - k_4\chi^4 - \frac{1}{4}\chi^4 ln\frac{\chi^4}{\chi_0^4} + \frac{\delta}{3}\chi^4 ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0}$$

$$\mathcal{L}_{vec} = \frac{1}{2}\frac{\chi^2}{\chi_0^2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2\right) + g_4^4(\omega^4 + 6\omega^2\rho^2 + \rho^4)$$

$$\mathcal{L}_{SB} = -\frac{\chi^2}{\chi_0^2}\left[m_\pi^2 f_\pi\sigma + \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right]$$

In order to determine the supernova matter properties, the thermodynamical potential of the grand canonical ensemble has to be solved. It is defined as

$$\frac{\Omega}{V} = -\mathcal{L}_{vec} - \mathcal{L}_{SB} - \mathcal{L}_{scal} - \nu_{vac} - \sum_i \frac{\gamma_i}{(2\pi)^3} \times \int d^3k[\,E_i^*(k) - \mu_i^*]$$

The energy density and the pressure follow from the Gibbs-Duhem relation,

$$\epsilon = \Omega/V + \mu_i\rho^i$$

and $$P = -\Omega/V$$

Results and Discussion

After the calculation of equation of state, we investigate the properties of supernova matter. Some conditions are used to investigate the properties of supernova matter.

In a supernova core, neutrinos are trapped and form an ideal Fermi-Dirac gas. Thus, the weak process is

$$p + e^- \leftrightarrow n + \nu_e$$

The chemical equilibrium requires

$$\mu_p + \mu_e = \mu_n + \mu_\nu$$

The second condition is charge neutrality,

$$\rho_e + \rho_\mu = \rho_p,$$

where ρ_e and ρ_μ are the electron and muon number densities respectively.

The third condition is to fix proton concentration,

$$Y_p = \frac{1}{2}\left(\frac{\rho_p - \rho_n}{\rho_p + \rho_n}\right)$$

where ρ_p and ρ_n are the proton and neutron number densities respectively.

The fourth condition is to add lepton concentration in stellar matter.

$$Y_l = \frac{\rho_e + \rho_\nu}{\rho} \equiv Y_e + Y_\nu$$

The pressure P versus the energy density ε is displayed in Figure.1 for nuclear matter varying proton fraction (Y_p = 0.1 to 0.5). The EOS is found to be considerably softer for the bigger Y_p in the effective chiral model compared with the EOS for the smaller Y_p. The pressure is the same until the energy density of ~ 170 MeV/fm^3 for all Y_p. Above this energy density the pressure is higher as the bigger value of Y_p. For Y_p = 0.5, the EOS is considerably softer than the EOS with smaller value of Y_p.

Figure.2 shows the energy per baryon as a function of baryon density for varying proton fraction calculated in the effective chiral model. With increasing proton fraction the energy per baryon decreases around saturation density. If the baryon density is larger than the saturation density, it is found that the energy per baryon rapidly increases for the larger proton fractions.

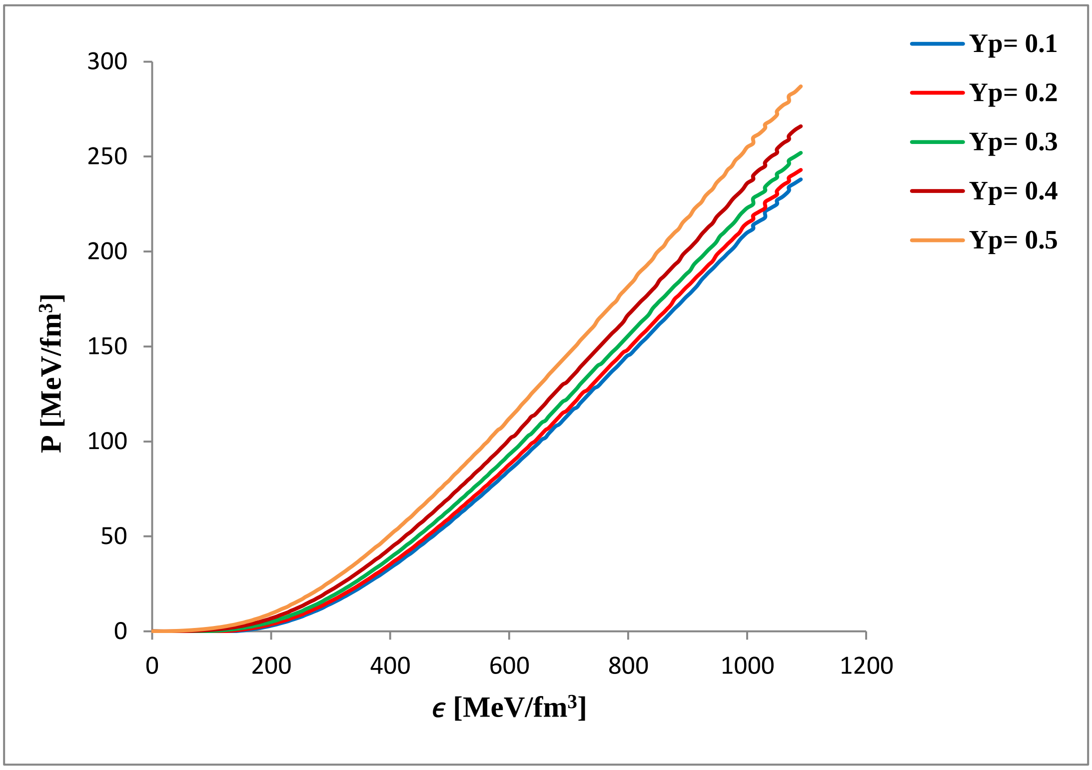

Figure 1 The equation of state (i.e. the pressure vs. energy density) with various proton fractions (Y_p).

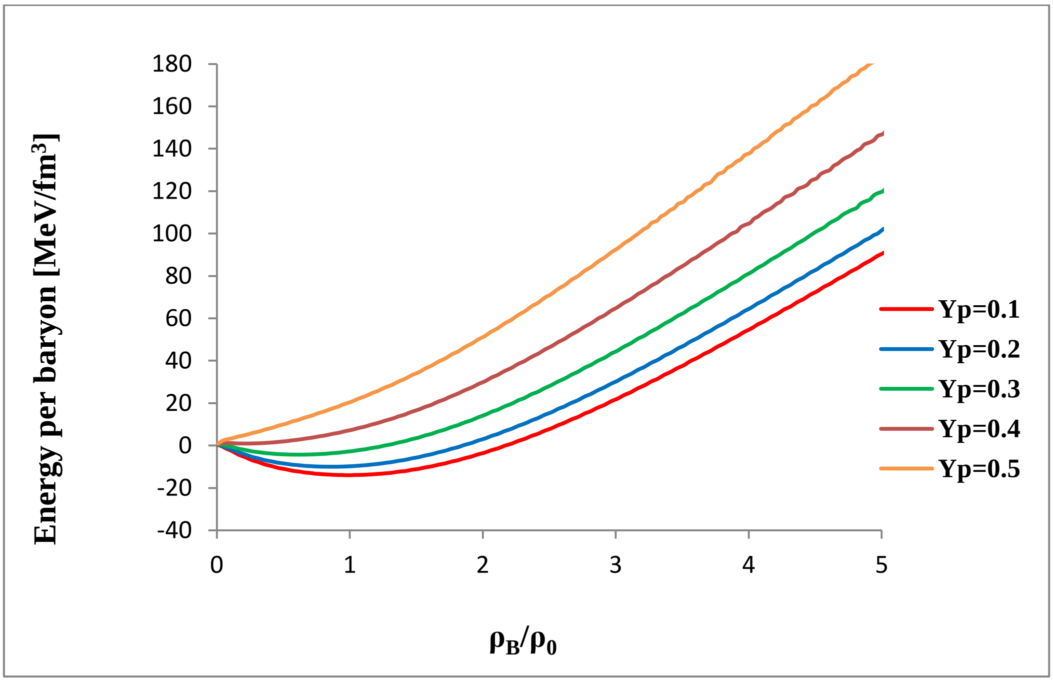

Figure 2 Binding energy per baryon vs. baryon density with Y_p=0.1 to 0.5.

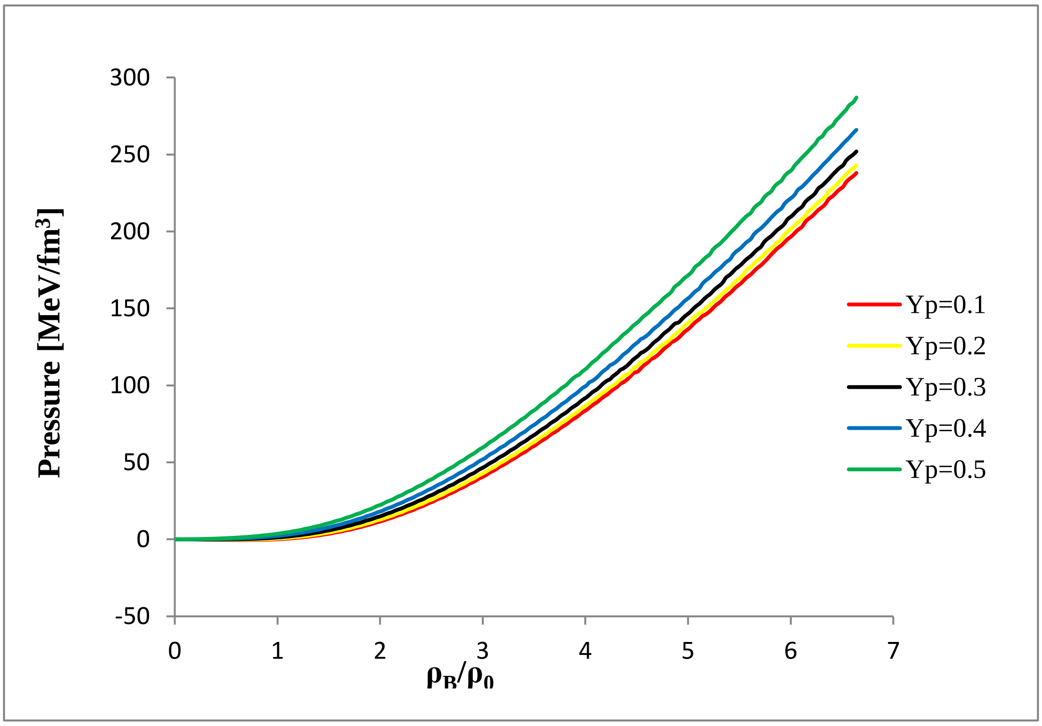

Figure 3 The pressure P as a function of baryon density ρ_B/ρ_0.

The pressure p as a function of the baryon density varying proton fraction is plotted in Figure 4.3. The pressure is the same around the saturation density. The pressure is nearly the same for the case of Y_p = 0.1 and Y_p = 0.2 although the pressure is higher when Y_p > 0.2.

In order to calculate the equation of state with lepton fractions, the extra chemical potential has added. The external chemical potential is used to fix the electron (plus muon) number. The pressure P versus the energy density ε is displayed in Figure.4 for supernova matter varying electron fraction (Y_e = 0.13 to 0.5). In Figure.1, Stellar matter is charge neutral, so if a fraction Y_e=0.2, Y_p=0.2 in order to have as many electrons and protons to achieve charge neutrality. In Figure.4, we have calculated star matter, which includes also leptons, i.e. electrons. The contributions of electrons are added to the total energy, thus energy per baryon are increased. The more neutrinos are emitted through the electron capture on protons. As a result, the bigger electron fraction (Y_e), the softer equation of state is formed.

The pressure p as a function of the baryon density varying electron fraction is plotted in Figure 5. When there are bigger Y_e, the pressure increases rapidly as the baryon density increases.

Figure 6 shows the energy per baryon as a function of baryon density for varying electron fraction calculated in the effective chiral model. According to added the lepton fraction, the binding energy per baryon are increased significantly. Due to the neutrino emission through the electron capture on protons, the star becomes neutron rich, leading to a cold neutron star.

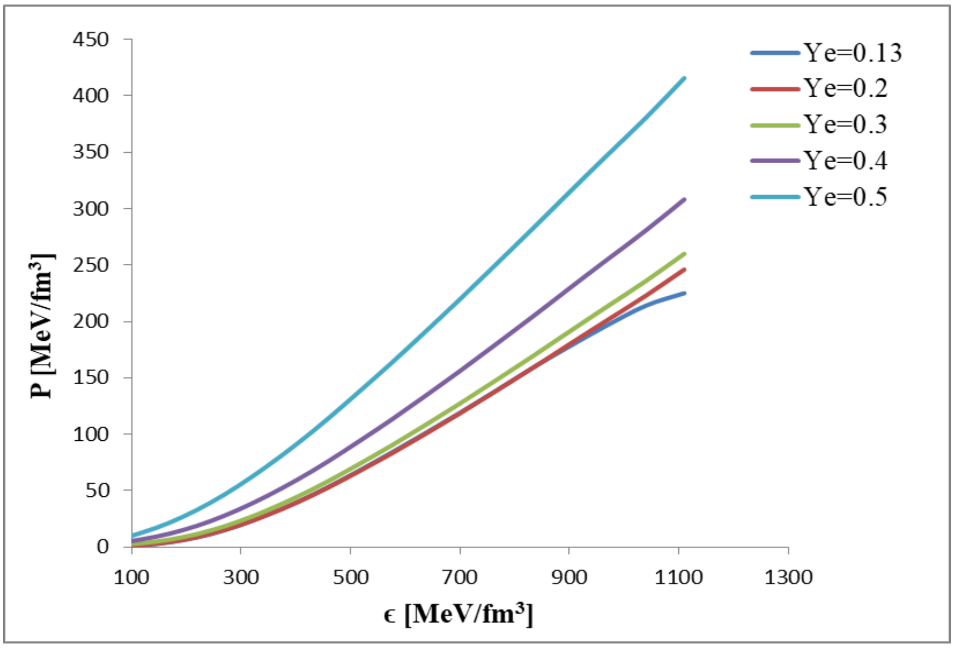

Figure 4 The equation of state (i.e. the pressure vs. energy density) with various electron fractions (Y_e=0.13 to 0.5).

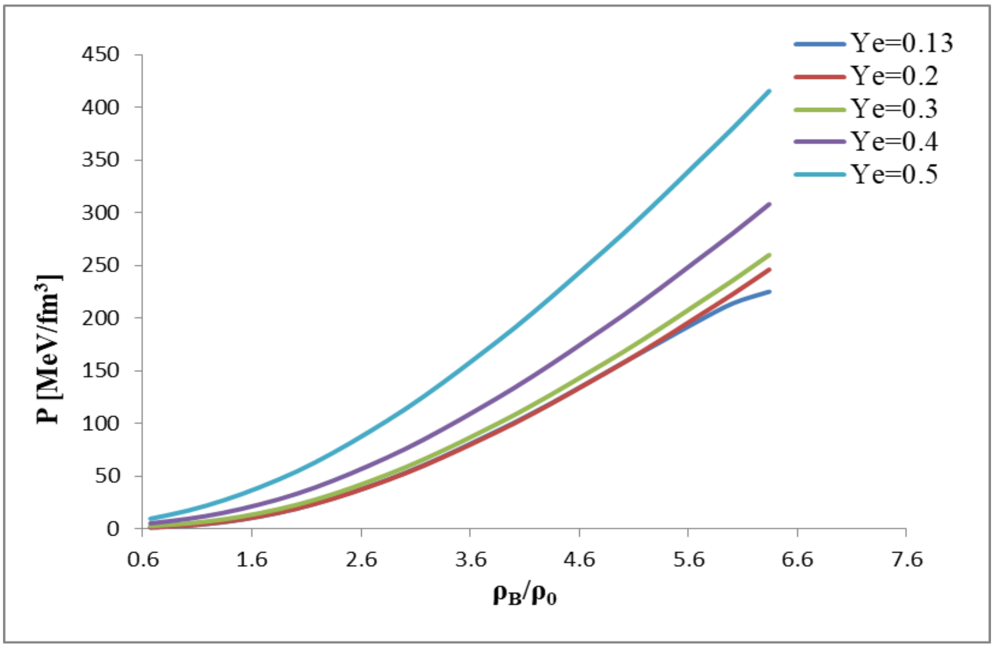

Figure 5 The pressure P as a function of baryon density ρ_B/ρ_0 (Ye=0.13 to 0.5).

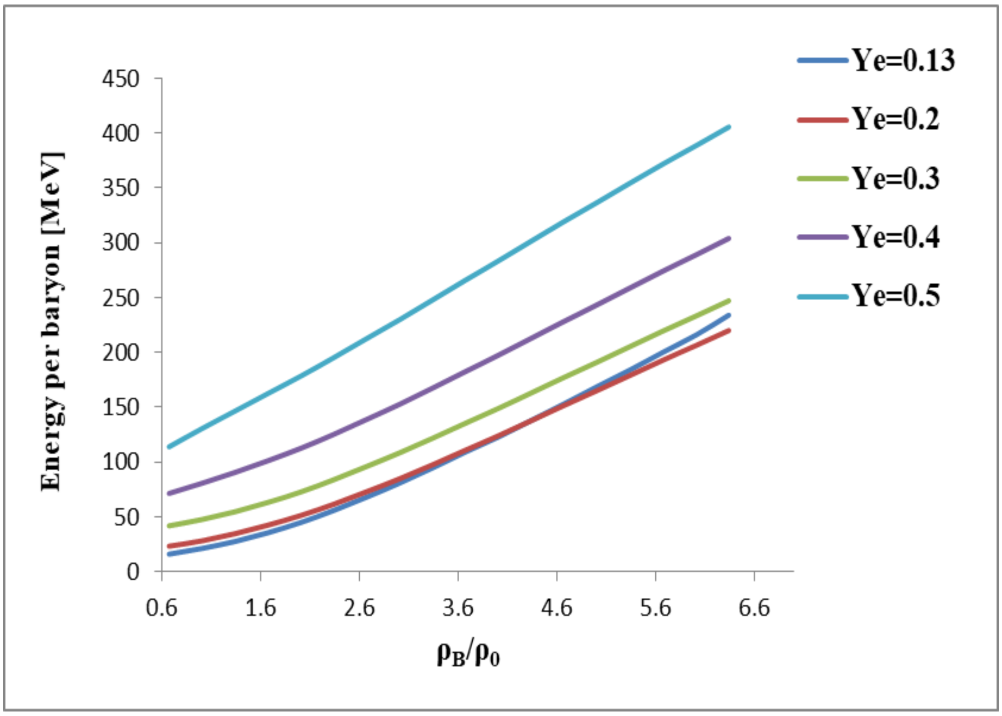

Figure 6 Binding energy per baryon vs. baryon density with Ye=0.13 to 0.5.

Conclusion

In this article, we have presented set of EOS table covering the wide range of baryon density, various proton fraction Y_p, electron fraction Y_e and temperature T = 1MeV for the use of core collapse supernova simulations. In our EOS, we have compared the equations of state with and without electron concentrations. The equation of state without electron fractions, Y_e, stellar matter is charge neutral, so if a fraction Y_e=0.2, Y_p=0.2 in order to have as many electrons and protons to achieve charge neutrality. The nuclear matter likes to be isospin symmetric if one neglects Coulomb interaction. The EOS is found to be considerably softer for the bigger Y_p in the effective chiral model compared with the EOS for the smaller Y_p. The EOS with electron fractions, Y_e, the extra chemical potential is added, the energy per baryon is increased significantly. This is due to the neutrino emission through the electron capture on protons. In addition to the properties of dense matter, it is mandatory to provide the rates of neutrino reactions with the dense matter in the supernova core. The reaction rates depend on the energy and the structure of nuclei. Therefore, implementing the neutrino reaction rates is also a challenging task for the numerical simulation of supernova explosion.

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