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DETERMINATION OF DENSITY DISTRIBUTION AND ROOT MEAN SQUARE RADII FOR LIGHT NUCLEI USING WOOD-SAXON AND HARMONIC OSCILLATOR POTENTIAL

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Abstract

The main purpose of this study is to investigate the nuclear charge density distribution corresponding proton, charge, neutron, matter and root mean square radii for light nuclei such as ^{4}He, ^{12}C and ^{16}O. A detail series of calculations were carried out using the single particle reduced radial wave function and radial wave function by **Numerov method**. The density distributions and root mean square radii of neutron, proton, matter and separation energies for stable nuclei have been calculated by using Woods-Saxon potential and Harmonic- Oscillator potential with **FORTRAN LANGUAGES**. In order to compare the calculated results of density distributions, root mean square radii and separation energies for light nuclei with the experimental data and other literatures.

Keywords: Stable nuclei, density distributions, root mean square radii, separation energies, Woods-Saxon and Harmonic-Oscillator potential.

Introduction

The radial distribution of nuclear charge, matter and the spatial extant of atomic nuclei have received great attention. They are important to explore sizes and shapes of nuclei, besides to test the validity of the nuclear single-particle wave functions used especially in density folding models. The study of nuclear neutron and proton density distributions are essential quantities in understanding both nuclear structure and reactions.

The distribution of charge density and charge radii can be measured accurately from high energy electron elastic scattering, but such scattering is so far limited to stable or long-lived isotopes, besides, the measurements of muonic X-rays transitions are precise method for determining charge radii. Information about matter density distributions are obtained from experiments on elastic hadron scattering. Harmonic Oscillator potential is not accurate to describe the nuclear central confining potential because the potential continues to give a contribution even for much larger r and does not become zero, besides the radial wave functions obtained from HO have a Gaussian fall-off behaviour at large r which does not reproduce the correct exponential tail.

Elton and Swift generated wave functions in a parameterized single-particle local potential and adjusted the parameters so as to fit the shape of the wave functions to elastic electron scattering data and the eigen energy to the proton separation energies in the 1p and 2s-1d shell nuclei. Gibson et. al. studied the ground state of the ^{4}He nucleus using the single-particle phenomenological model. Wave functions were generated from a potential whose parameters are chosen to reproduce the correct neutron separation energy. The proton separation energy, electron scattering form factors were then calculated. Brown et. al. described a new method of calculating nuclear charge and matter distributions which is complementary to the Hartree-Fock method talking into account shell method configuration mixing. Brown et. al. calculated the rms radii of valence orbits in the tin isotopes using the single-particle potential model. Lojewski et. al.

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used realistic single-particle WS potential to evaluate the mean-square charge radii of even-even nuclei. The various parameterizations of the WS potential were examined. Schwierz et. al. established a new parameterization for the WS potential. Its six parameters are fitted to single-particle spectra around doubly magic nuclides and experimental charge radii. In the eigenvalues have been calculated using Numerov method for a Sturm-Liouville problem defined with the boundary values. Recently, Arkan has been calculated the nuclear charge density distributions, elastic charge form factors and rms charge, proton, neutron and matter radii for $^{4}\mathrm{He}$, $^{12}\mathrm{C}$, and $^{16}\mathrm{O}$ nuclei using WS and HO radial waves function using different well potential depths for WS potential for each subshell.

The Harmonic-Oscillator Potential

We know our experience with classical mechanics that a particle of mass m subject to the linear restoring force F(x)= -kx ,where k is the force constant, results in one-dimensional simple harmonic motion with an oscillation frequency $\omega = \sqrt{k/m}$. The potential that the particle moves in is quadratic $V(x) = kx^2/2$, and so in this case the potential has a minimum at position x=0. The idea that a quadratic potential may be used to describe a local minimum in an otherwise more complex potential turns out to be a very useful concept in both classical and quantum mechanics. It is assumed that the nucleons moves in an average harmonic oscillator potential given by

$$V_{HO} = \frac{1}{2} m\omega^2 r^2 \tag{1}$$

Where m is the mass of the nucleons and ω is the oscillator frequency.

Woods-Saxon Potential

For the local potential, the Woods-Saxon shape is used in the compact form shown below

$$v(r) = v_{cent}(r) + v_{so}(r) + v_c(r) \tag{2}$$

$$v(r) = \frac{-U_0}{\left(1+e^{\left(\frac{r-R}{a}\right)}\right)} + \left(\frac{\hbar}{m_\pi}\right)^2 \frac{1}{r}\frac{U_{so}}{a_{so}} \frac{e^{\left(\frac{r-R_{SO}}{a_{SO}}\right)}}{\left(1+e^{\left(\frac{r-R_{SO}}{a_{SO}}\right)}\right)} \left\langle \hat{\ell}, \hat{\sigma} \right\rangle + v_c(r) \tag{3}$$

The Woods-Saxon potential equation contains three parts, they are central potential, spin-orbit potential and coulomb potential.

Central Potential

The central potential equation can be written as

$$v_{cent}(r) = \frac{-U_0}{\left(1+e^{\left(\frac{r-R}{a}\right)}\right)} \tag{4}$$

Where,

$\nu_{cent}(r)$ = the central potential

U_0 = the strength or depth of the central potential

a = the diffuseness of central potential

Spin-Orbit Potential

Spin-orbit splitting of nuclei is one of the main factors which determine nuclear structure in nuclei both near and far from the closed shells. Spin and orbit refer to the attributes of a single nucleon moving in an averaged potential well. Spin-Orbit splitting in atoms is due to the magnetic interaction of the magnetic dipole moment of the electron spin and the magnetic field experienced by the electron in its rest frame as it moves through the Coulomb field of the nucleus. A nucleon moving in the central potential of the nucleus with orbital angular momentum ℓ, spin s and total angular momentum j,

$$\vec{\ell}.\vec{s} = \frac{1}{2}\ell \qquad \text{for } j = \ell + \frac{1}{2} \tag{5}$$

$$\vec{\ell}.\vec{s} = -\frac{1}{2}(\ell + 1) \qquad \text{for } j = \ell - \frac{1}{2} \tag{6}$$

The splitting due to the spin-orbit interaction is larger for higher values of orbital angular momentum, and can consequently produce level crossing. For large angular momentum (ℓ), the splitting of any two neighbouring degenerate levels can shift the j= ℓ+1/2 state of the initially lower level to lie above the j= ℓ-1/2 state of the previously higher level.

$$\nu_{so}(r) = -2\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{U_{so}}{r}\frac{d}{dr}\frac{1}{\left(1+e^{\left(\frac{r-R_{SO}}{a_{SO}}\right)}\right)}\left\langle\hat{\ell},\hat{\sigma}\right\rangle = 2\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{U_{so}}{ra}\frac{e^{\left(\frac{r-R_{SO}}{a_{SO}}\right)}}{\left(1+e^{\left(\frac{r-R_{SO}}{a_{SO}}\right)}\right)^2}\left\langle\hat{\ell},\hat{\sigma}\right\rangle \tag{7}$$

Where,

$$\left(\frac{\hbar}{m_{\pi c}}\right)^2 \approx 2.0 \text{ fm}^2$$

U_{so} = the strength or depth of the spin-orbit potential

a_{so} = the diffuseness of central potential

R_{so} = the radius parameter of spin-orbit

$\hat{\ell}$ = angular momentum

$\hat{\sigma}$ = spin operator

Coulomb Potential

The Coulomb potential part of WS potential

$$\nu_c(r) = \begin{cases} (Z-1)\dfrac{e^2}{r} & \text{if} \quad r > R_c \\ \dfrac{(Z-1)e^2}{2R}\left[3 - \dfrac{r^2}{R^2}\right] & \text{if} \quad r < R_c \end{cases} \tag{8}$$

Where $\nu_c(r)$ = 0 for neutrons, with e^2 = 1.44MeV

Configuration for light nuclei (^{4}He, ^{12}C and ^{16}O)

The nuclide contains protons and neutrons. According to shell configuration, the terms of protons and neutrons are represented by the following equation. The configuration of protons is $\left(1s_{\frac{1}{2}}\right)^2$ and the configuration of neutrons is $\left(1s_{\frac{1}{2}}\right)^2$ for ^{2}He. The configuration of protons is $\left(1s_{\frac{1}{2}}\right)^2$ $\left(1p_{\frac{3}{2}}\right)^4$ the configuration of neutrons is $\left(1s_{\frac{1}{2}}\right)^2\left(1p_{\frac{3}{2}}\right)^4$ for ^{12}C. The configuration of protons is $\left(1s_{\frac{1}{2}}\right)^2\left(1p_{\frac{3}{2}}\right)^4\left(1p_{\frac{1}{2}}\right)^2$ The configuration of neutrons is $\left(1s_{\frac{1}{2}}\right)^2\left(1p_{\frac{3}{2}}\right)^4\left(1p_{\frac{1}{2}}\right)^2$ for ^{16}O.

The reduced radial wave function of two body Schrödinger equation

The Schrodinger wave equation can be expressed as the following.

$$\left(\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} - V(r) - \ell(\ell+1)\frac{\hbar^2}{2\mu r^2} + E_{nlj}\right)R_{nlj}(r) = 0 \tag{9}$$

This equation is the Schrödinger equation for the single-particle radial wave function .Where μ is the reduce mass of the core (A-1) and single nucleon, m is the mass of nucleon, A is the atomic mass, E_{nlj} is the single nucleon separation energy, $R_{nlj}(r)$ is the radial eigenfunction of the system and n, ℓ and j are the principal, orbital angular and total angular quantum numbers. Single particle energy level of a nucleon moving in a potential well is determined by using the Numerov method. The Schrödinger radial equation is given by

Where, $R_{nlj} = \dfrac{U(r)}{r}$ is the radial wave function. (10)

Separation Energies, Density Distributions and Roots-Mean Square Radii of the Light Nuclei

The energy required to remove the last neutron from the nucleus is called the separation energy of the last neutron. If a neutron is removed from a nuclide (Z, N), a nuclide (Z, N-1) is obtained. The energy needed for removal the last neutron is the difference in binding energies between these two nuclides. According to the shell model the last neutron separation energy is the binding energy of the outermost filled level of a nucleus.For light nuclei, the calculated orbitals are in angular momentum states. In $^4_2$He, $1S_{1/2}$ orbit is found. In $^{12}_6$C nuclei, $1s_{1/2}$, $1P_{3/2}$

and in ${}^{16}_{8}O$, $1s_{1/2}$, $1P_{3/2}$, $1P_{1/2}$ orbits are found. This kind of single particle orbits cannot be seen in ordinary nuclei. There are calculated the parameters for central potential, spin-orbit potential, centrifugal potential and Coulomb's potential term with Woods-Saxon potential. The values are shown in Table (1).

To understand the structure of the nuclei we have determined the root-mean square radius and charge density distribution. The root-mean square radius of the nucleus $\langle r^2 \rangle^{\frac{1}{2}}$ is defined as

$$\langle r^2 \rangle^{\frac{1}{2}}_{n,p,ch,m} = \sqrt{\frac{4\pi}{X} \int U^*(r) U(r) r^2 dr} \tag{11}$$

The calculated results of physical quantities are shown in table (2). The density distribution of the nuclei is defined as the following equation,

$$\rho(r) = \int |R_{nl}|^2 r^2 dr = \int \left| \frac{U(r)}{r} \right|^2 r^2 dr \tag{12}$$

Therefore, the point density distribution of neutrons, protons and matter can written respectively as;

$$\rho_{n,p\,or\,m}(r) = \frac{1}{4\pi} \sum_{n\ell j} X^{n\ell j}_{n,p\,or\,m} \left| R_{n\ell j}(r) \right|^2 \tag{13}$$

Where $X^{n\ell j}_{n,p\,or\,m}$ represented the number of neutrons, protons or nucleons in the $n\ell j$ -subshell.

Results and Discussion

In this research work, the nuclear shell model is used to calculate the charge density distribution and rms radii for ^4He, ^{12}C and ^{16}O nuclei. The harmonic oscillator and Woods-Saxon potentials are used to regenerate the radial wave function and reduced radial wave function. It was calculated the CDD' s by using reduce radial wave function and also rms radii by using reduced radial wave function for ^4He, ^{12}C and ^{16}O nuclei. In the present work, light nuclei have the following configurations, $(1S_{1/2})^2$ state are used for ^4He, $(1S_{1/2})^2$, $(1P_{3/2})^4$ for ^{12}C, $(1S_{1/2})^2$, $(1P_{3/2})4$ and $(1P_{1/2})^2$ for ^{16}O nuclei. The diffuseness parameters chosen for WS potential of light nuclei as shown in Table (1). The results for the calculated single nucleon separation energies for ^4He, ^{12}C and ^{16}O nuclei of different sub-shells are shown in Table (2). The experimental charge rms radii and calculated rms radii for neutron and proton in light nuclei ^4He, ^{12}C and ^{16}O are displayed in Table (3). The calculated charge density distributions are depicted in Fig (1) to Fig (3) for ^4He, ^{12}C and ^{16}O nuclei. Comparison of charge density distributions are illustrated in Fig (4) for investigated nuclei.

Table 1 The WS parameters U_o, $U_{s.o}$, a_o, $a_{s.0}$, r_o, $r_{s.o}$ and R_c for ^{4}He, ^{12}C and ^{16}O nuclei.

Nucleus	nl_j	U_o(MeV)	$U_{s.o}$(MeV)	a_o(fm)	$a_{s.o}$(fm)	r_o(fm)	$r_{s.o}$(fm)	R_c(fm)
N	$1s_{1/2}$	56.70	15.0	0.1	0.01	1.350	1.350	0.0
P	$1s_{1/2}$	56.53	15.0	0.01	0.01	1.333	1.333	0.0
^{12}C								
N	$1s_{1/2}$	59.76	15.0	0.527	0.527	1.236	1.236	0.0
	$1p_{3/2}$	59.10	15.0	0.527	0.527	1.236	1.236	0.0
P	$1s_{1/2}$	60.05	15.0	0.518	0.518	1.230	1.230	1.23
	$1p_{3/2}$	59.21	15.0	0.518	0.518	1.230	1.230	1.23
^{16}O								
N	$1s_{1/2}$	51.0827	15.0	0.5	0.5	1.375	1.375	0.0
	$1p_{3/2}$	50.1804	15.0	0.5	0.5	1.375	1.375	0.0
	$1p_{1/2}$	52.4350	15.0	0.5	0.5	1.375	1.375	0.0
P	$1s_{1/2}$	50.6656	15.0	0.5	0.5	1.375	1.375	1.375
	$1p_{3/2}$	50.3532	15.0	0.5	0.5	1.375	1.375	1.375
	$1p_{1/2}$	52.4822	15.0	0.5	0.5	1.375	1.375	1.375

Table 2 Calculated ($E_{calculted}$) and ($E_{experiment}$) single neutron and proton separation energies for nlj subshells for ^{4}He, ^{12}C and ^{16}O.

^{4}He	nl_j	$E_{experiment}$(MeV)	$E_{calculated}$(MeV)
n	$1s_{1/2}$	20.5776	20.7538
p	$1s_{1/2}$	19.8139	19.7523
^{12}C			
n	1s1/2	34.03	34.05
	1p3/2	18.72	18.74
p	$1s_{1/2}$	30.9	30.65
	$1p_{3/2}$	15.75	15.70
^{16}O			
n	1s1/2	34.03	34.04
	1p3/2	21.81	21.83
	1p1/2	15.65	15.68
p	1s1/2	29.81	29.66
	$1p_{3/2}$	18.44	18.34
	$1p_{1/2}$	12.11	11.90

Table 3 The value of rms radii in (fm) with corresponding available literature data.

Nucleus	literature $\langle r^2\rangle_n^{\frac{1}{2}}$	Exp. $\langle r^2\rangle_{ch}^{\frac{1}{2}}$	Calculated $\langle r^2\rangle_p^{1/2}$	Calculated $\langle r^2\rangle_n^{\frac{1}{2}}$
^{4}He	WS:1.704	1.676	WS:1.714	WS:1.704
	HO:1.659		HO:1.577	HO:1.676
^{12}C	WS:2.316	2.464	WS:2.501	WS:2.399
	HO:2.287		HO:2.390	HO:2.291
^{16}O	WS:2.589	2.737	WS:2.698	WS:2.578
	HO:2.458		HO:2.670	HO:2.412

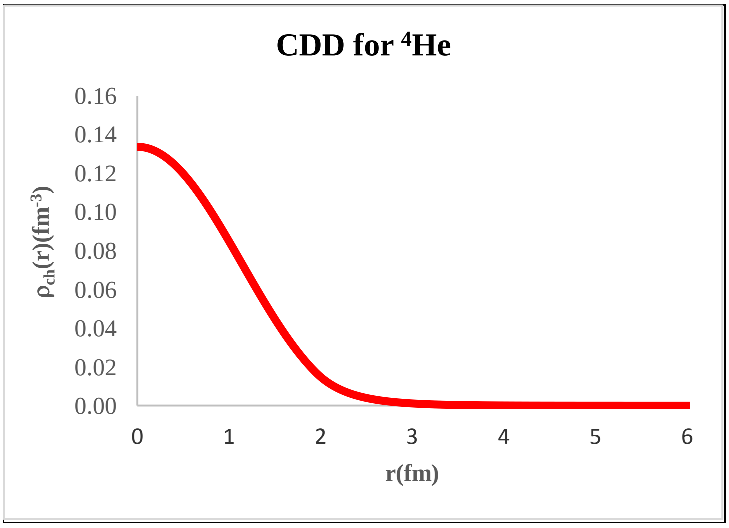

Figure 1 CDD's for nlj state in ^{4}He nucleus.

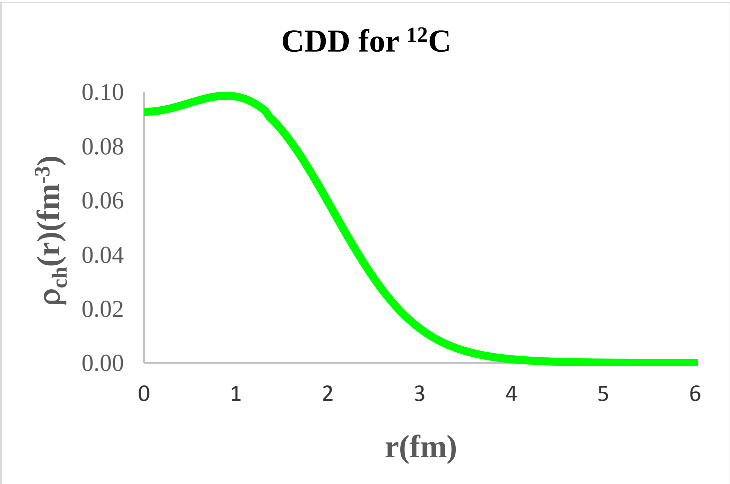

Figure 2 CDD's for nlj state in ^{12}C nucleus with WS potential.

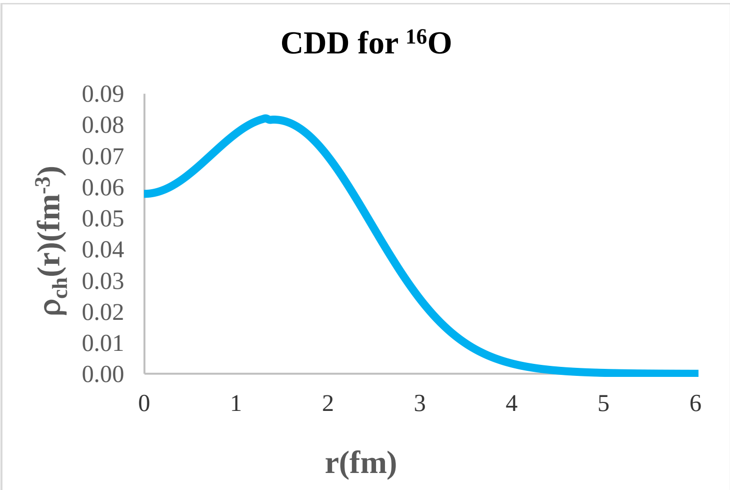

Figure 3 CDD's for nlj state in ^{16}O nucleus with WS potential.

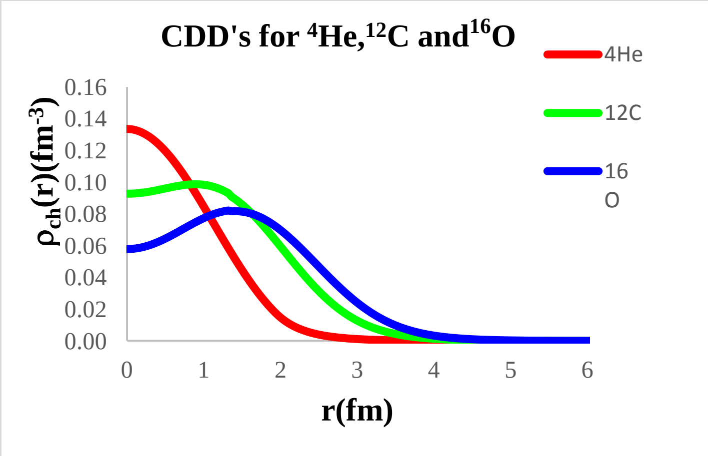

Figure 4 Comparison of CDD's for nlj state in ^{4}He ,^{12}C and^{16}O nucleus with WS potential.

Conclusions

For the calculated CDD's and root mean square radii, the result of Woods-Saxon potential are much better than harmonic oscillator potential in ^{4}He, ^{12}C and ^{16}O nuclei. The calculated results of separation energies for consider nuclei obtained by WS potential are in very good agreement with experimental data. It can be seen that the tightly bound in ^{4}He nucleus and loosely bound at the surface of ^{16}O nucleus

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